DESTination RAIL – Decision Support Tool for Rail Infrastructure Managers
Project Reference: 636285
H2020-MG 2014-2015
Innovations and Networks Executive Agency
Project Duration: 1 May 2015–31 April 2018

A network flow model to determine optimal intervention programs for railway infrastructure networks

Authors

*** Marcel Burkhalter, Prof. Dr. Bryan T. Adey, Natalia Papathanasiou**

*Corresponding author: Marcel Burkhalter, burkhalter@ibi.baug.ethz.ch

Date: April,2018

Dissemination level: (PU, PP, RE, CO): PU

This project has received funding from the European Union's Horizon 2020 research and innovation program under grant agreement No 636285

This project is funded by the European Union

DOCUMENT HISTORY

Number	Date	Author(s)	Comments
01	31/03/2018	Marcel Burkhlater, Prof. Dr. Bryan T. Adey, Natalia Papathanasiou	Deliverable for first review
02	18/04/2018	Marcel Burkhlater, Prof. Dr. Bryan T. Adey, Natalia Papathanasiou	Final deliverable

Executive Summary

In order for a railway network to be used as intended, the railway infrastructure has to function as intended. This infrastructure, however, deteriorates and will fail over time if interventions are not executed. The probability of occurrence of failures and their consequences is referred to as risk.

The main task of a railway infrastructure manager is to execute interventions to reduce this risk. These preventive interventions, however, do not come without costs, e.g. intervention costs incurred by the railway infrastructure manager and user costs due to the restriction of train movements. Ideally, a railway infrastructure manager will determine the intervention program that will provide the maximum net benefit, taking into consideration the costs of all relevant stakeholders.

In the determination of an optimal intervention program for an infrastructure network requires taking into consideration how infrastructure objects are connected within the network. This is necessary because executing interventions in groups can result in reductions of the both owner costs, e.g. costs of setting up a work zone, and users, e.g. costs of restricting train movements. If the possible cost reductions are explicitly taken into consideration, infrastructure managers may be able to do away with their default assumption that they should execute interventions on railway infrastructure objects at night whenever possible, due to the low or inexistent traffic at this time, as other time windows could be beneficial, as they would be able to work uninterrupted for longer periods of time.

This report contains a model to be used to determine optimal intervention programs for objects of different types in a railway infrastructure network, taking into consideration how the objects in the network are connected. The optimal intervention program is the one which has the maximum net-benefit. The report is the deliverable associated with task 3.3 of the DESTination Rail research project. The estimation of the risk used in this report was estimated using the risk assessment methodology developed in task 3.2 of DESTination Rail. The optimal intervention program is determined using a constrained multilayer network flow model, which can be mathematically formulated as a mixed integer linear problem. The model consists of an intervention layer, where the interventions included in the intervention program are selected, and traffic state layers, where the flow models the duration this particular traffic state is required to execute the interventions. The network flow model considers economical and topological dependencies between the objects. Economical dependencies refer to the possibility to reduce the owner costs, while topological dependencies refer to the possibility to reduce the overall duration of traffic disturbance. The constraints include the flow conservation constraints, organisational constraints, i.e. a budget limitation, and structural constraints that represent structural dependencies between objects, i.e. the track on a bridge.

The model is illustrated using a part of the Irish railway network located in Dublin, Ireland, which consists of approximately 2’200 meters of double track line, 23 switches and 39 bridges. The comparison of the intervention program developed by the network model presented with the intervention programs developed by a reduced exhaustive search and an approach using simplified decision rules shows that the network model can be used to determine the optimal intervention program within reasonable time. Challenges are, however, expected if the network is expanded.

Table of Contents

Executive Summary 3
Table of Contents 4
List of Figures 6
List of Tables 7
1 Introduction 9
2 Literature review 10
2.1 Intervention planning on railway infrastructure 10
2.2 Other infrastructure 13
3 Background 15
3.1 Infrastructure objects 15
3.2 Topological network breakdown 17
3.3 Interventions 17
3.4 Network dependencies 19
3.5 Time window of intervention execution 22
4 Conceptual model 24
4.1 Objective function 24
4.2 Constraint types 26
4.3 Structure 26
5 General mathematical model 37
5.1 Objective 37
5.2 Flow conservation constraint 37
5.3 Duration transformation constraint 37
5.4 Exclusivity constraint 38
5.5 Equal flow constraint 38
5.6 Organisational constraint 38
5.7 Structural constraint 38
6 Example 39
6.1 Example situation 39
6.2 Approaches to develop an optimal intervention program 58
6.3 Results 64
6.4 Discussion 68
7 Summary and Conclusions 70
8 References 71

Annex A Example area ... 76
Annex B Summarising object table ... 77
Annex C Results of exhaustive search ... 81
Annex D Results of decision rules ... 84
Annex E Results of network flow model ... 87

List of Figures

Figure 1. Railway network breakdown 17
Figure 2. Network dependencies 20
Figure 3. Effect of time windows 23
Figure 4. Cost of an intervention program 25
Figure 5. Simplified illustration of the network flow model 27
Figure 6. Illustration of the intervention layer 31
Figure 7. Illustration of the traffic state layers 33
Figure 8. Relationship between the traffic state duration and the individual intervention durations considering different intervention types 34
Figure 9. Network structure of a traffic state layer 34
Figure 10. Area of the example (map source: OpenStreetMap Contributors, 2017) 40
Figure 11. Network layout of the example area 41
Figure 12. Objects in the example including their current state 43
Figure 13. Objects in the example 76
Figure 14. Current state of the objects 76
Figure 15. Intervention program by exhaustive search without a budget limitation 83
Figure 16. Intervention program by decision rules without a budget limitation 86
Figure 17. Intervention program by decision rules with a budget limitation of 4 Mio. Euros 86
Figure 18. Intervention program of the network flow model without a budget limitation 89
Figure 19. Intervention program of the network flow model with a budget limitation of 4 Mio. Euros 89

List of Tables

Table 1. Object categories in a railway infrastructure network - adapted from (European Union, 2012) 16
Table 2. Intervention types for railway infrastructures 19
Table 3. Node types per layer 29
Table 4. Edge types (part 1) 30
Table 5. Edge types (part 2) 31
Table 6. Objects included in the example 42
Table 7. Track segments 44
Table 8. Switches 45
Table 9. Bridges 46
Table 10. Risk related to track segments [Euros] 47
Table 11. Risk related to switches [Euros] 48
Table 12. Risk related to bridges [Euros] 49
Table 13. Interventions for tracks 50
Table 14. Interventions for switches 50
Table 15. Interventions for bridge 50
Table 16. Structural dependencies 52
Table 17. Economical dependencies 53
Table 18. Topological dependencies 54
Table 19. Time windows considered 54
Table 20. Losses in level of service and user costs due to closed track segments 55
Table 21. Estimation of passengers per train 56
Table 22. Delay calculation for closure of segment T_1 during the week. 57
Table 23. Total number of possible intervention programs 58
Table 24. Total number of possible intervention programs after possible reductions 59
Table 25. Rules and their influence on object categories 60
Table 26. Consideration of interventions due to decision rules 62
Table 27. Characteristics of the network flow model in the example 63
Table 28. Characteristics of the mixed integer linear program from the network flow model 64
Table 29. Intervention programs developed without a budget constraint 65
Table 30. Net benefit of the intervention programs developed without a budget constraint 66
Table 31. Intervention programs developed with a budget limitation of 4 Mio euros 66
Table 32. Net benefit of the intervention programs developed with a budget limitation of 4 Mio euros 67
Table 33. Traffic states of the intervention program for the network flow model 67
Table 34. Computation time for the different models 69
Table 35. Summary of track objects in the example 77
Table 36. Summary of switch objects in the example 78
Table 37. Summary of bridge objects in the example (part 1) 79
Table 38. Summary of bridge objects in the example (part 2) 80
Table 39. Intervention program developed by exhaustive search without a budget limitation 81
Table 40. Traffic states of the intervention program for the exhaustive search 82
Table 41. Intervention program developed by the decision rules without a budget limitation 84

Table 42. Intervention program developed by decision rules with a budget limitation of 4 Mio. Euros 85
Table 43. Traffic states of the intervention program for the decision rule approach 85
Table 44. Intervention program developed with the network flow model without a budget limitation 87
Table 45. Intervention program developed with the network flow model with a budget limitation of 4 Mio. Euros 88
Table 46. Traffic states of the intervention program for the network flow model 88

1 Introduction

A railway infrastructure network consists of objects from different categories (e.g. track, bridge, signalling) that have different characteristics and functions in the network. Executing interventions on these objects leads to losses in service, which can be minimized by either executing interventions within windows of time with low traffic or executing multiple interventions together in groups to reduce traffic disruption. While the benefit of grouping interventions within the same time windows, e.g. night, weekend, is well known, it is rarely considered when developing mathematical models to determine optimal intervention programs when objects of the same category, e.g. bridges, are being considered, and it is even rarer when objects of different object categories are considered. Most research until now has concentrated on developing optimal intervention strategies or programs for single objects categories. Little to no research has been conducted on grouping interventions on objects of different object categories while considering the net benefit of an intervention program as the reduction in risk through the execution of interventions and the costs incurred to the owner and user of the infrastructure while executing interventions. Railway infrastructure organisations usually develop intervention programs for each object category separately and, if at all, try to find synergies between the intervention programs in order to minimise required traffic restrictions. Neither the methodologies developed in research nor the approaches used in practice lead to optimal intervention programs of railway infrastructure networks.

In this report, a methodology is presented that enables the development of optimal intervention programs for railway infrastructure networks considering objects in different categories, dependencies between the objects themselves and between the objects and the network operation, the benefit of executing interventions in terms of risk reduction, and organisational constraints, i.e. a budget limitation. The methodology includes the consideration of economical, structural, and topological dependencies between objects, which are the most important system dependencies for developing intervention programs for railway infrastructure networks. Economical dependencies allow the consideration of possible differences between the costs of executing multiple interventions together compared to the summed costs of individual execution of the same interventions, e.g. shared fix costs. Structural dependencies refer to situations where the functionality of an object depends on the functionality of other objects, e.g. a track on a bridge. Topological dependencies refer to the relation between objects in respect of their functionality as an object, e.g. two bridges along single track route. Further, it is shown, how the results of the risk assessment process developed in task 3.2 of the Destination Rail project (Papathanasiou et al., 2016) can be used to develop optimal intervention programs. The benefit of an intervention program is defined as the difference in risk when no interventions are executed and the risk when the interventions in the intervention program are executed within one year. The risk, therefore, has to be estimated for the current state and the state after the interventions are executed.

The remainder of this report is structured as followed. Chapter 2 contains a literature review. Chapter 3 contains background technical information required to understand how the model is set up. Chapter 4 contains the description of the conceptual model. Chapter 5 contains the description of the mathematical model. Chapter 6 contains an example where the methodology and model are used to determine the optimal intervention program for an example network. Chapter 7 contains a summary and conclusions.

2 Literature review

2.1 Intervention planning on railway infrastructure

The planning of interventions on railway infrastructure has attracted increasing interest in research over the last two decades. The entire topic has been studied from different points of view, such as the development of intervention strategies and the development of intervention programs. Intervention strategies are the interventions to be executed on an object, which is considered in isolation, and which is in multiple possible states, e.g. no interventions should be executed on a track when it is in state 1 but tamping should be done if it is in state 2 and a replacement intervention if it is in state 3. Intervention programs are the interventions to be executed on multiple objects within a specified time period taking into consideration constraints, e.g. on spending, and possible synergies from executing interventions simultaneously on multiple objects, e.g. reduced user costs by reducing the time of traffic disruption. They are done at relatively general time periods such as years. Intervention schedules are the intervention programs done for short time periods, such as on daily or hourly schedule basis.

2.1.1 Intervention strategies

Intervention strategies are developed on the highest level of intervention planning. They have the objective to define under ideal conditions when interventions should be executed on objects, if they are assessed in isolation. Intervention strategies are used by infrastructure managers to develop intervention programs. The most important aspects of intervention strategies are the modelling of deterioration and the timing of interventions to determine the optimal life-cycle costs.

Research on the deterioration models and optimal intervention strategies is most extensive for tracks, for example Andrade & Teixeira (2012), Caetano & Teixeira (2013), Lyngby et al. (2008), and Zhao et al. (2006). The intervention strategies are usually investigated taking into consideration the owner costs due to the execution of interventions, e.g. tamping and track renewal, and the effect on the users due to a deteriorated track condition. Some researchers have incorporated both owner and user costs in the objective function, such as Lyngby et al. (2008) and Zhao et al. (2006) who did this by implementing a penalty cost based on the accumulated traffic load for losses in track quality, and through the consideration of a risk model such as the barrier, respectively. The barrier model considers the costs occurring when a failure detection mechanism – called barrier – fails. A barrier is, thereby, a part of the system that prevents or at least lowers the probability of a so-called "top-event" to happen. Others, such as Andrade & Teixeira (2012) and Caetano & Teixeira (2013), have formulated a bi-objective model to generate a Pareto optimal front between the owner costs and the effect on the user. This allowed infrastructure managers to define the optimal intervention strategy based on their own weighting between effects on the owner and the users.

Although most work has focused on single objects, some have attempted to take into consideration dependencies between objects when determining optimal intervention strategies. For example, Andrade & Teixeira (2012) investigated the optimal times to execute tamping and renewal interventions on multiple track segments connected serial on a route between two points, where they considered the user costs over the entire route and not based on the individual object. Others investigated single objects as multi-component systems. For example, Caetano & Teixeira (2013) considered a track segment as a multi-component system containing of rail, sleeper, and ballast, and determined the optimal intervention strategy for the track segment taking into consideration the deterioration speeds and costs of repairing each of the components.

Although bridges are important engineering structures in railway networks, due to their longevity, their relative infrequency and the relative infrequency of maintenance interventions on them when compared to track sections, and perhaps their uniqueness, there has been less research on the determination of intervention strategies on railway bridges. Moreu et al. (2016) and Yianni et al. (2016) are two of the most advanced works in this area. Moreu et al. (2016) determined the optimal intervention strategies that minimise the total costs, which were composed of intervention costs and operational costs due to deteriorated bridges. This work explicitly took into consideration the possibility of failure by using fragility curves to classify bridges into service level classes that are connected to operational costs. Yianni et al. (2016) have developed a Petri-Net model consisting of a deterioration module, an inspection module, and an intervention module. Compared with the former, this approach of developing management strategies, i.e. monitoring strategies and intervention strategies treating the bridge as an object composed by multiple elements.

Although there has been little work on the determination of optimal intervention strategies for railway bridges, there has been an abundance on road bridges (Liu & Frangopol, 2004; Lounis, 2006). Lounis (2006), for example, determined optimal intervention strategies for highway bridge decks using multi-objective optimisation making a trade-off between the intervention costs and user costs due to delays or detours, as well as risk, where risk was considered a combination of the consequences of failure and the deterioration of the bridge condition estimated using Bogdanoff's Cumulative Damage Model. A multi-objective index was used to transfer the generated Pareto front into an optimal result. The relative importance of the costs and risk was entered using weights.

Intervention strategy development models based on life cycle analysis have also been developed for other object categories, e.g. earthwork (Power et al., 2016), switches (Zwanenburg, 2009), signalling and safety equipment (Li, 2013; Morant et al., 2014), and power supply systems (Chen et al., 2013). Power et al (2016) presented an evidence-based earthwork asset management process to develop intervention strategies. This includes a risk-based prioritisation matrix for all earthwork assets, a quantitative determination of consequences of failure, the development of intervention types and their impact on the likelihood of failure, and a Whole Life Cycle Cost Decision Support Tool. Zwanenburg (2009) developed intervention strategies for railway switches and crossings considering detailed deterioration processes based on statistical analyses to retrieve the lifetime expectancy of complete railway switches and crossings and their respective components. Both Li (2013) and Morant et al. (2014) developed intervention strategies for signalling systems based on failure risk. While Li (2013) minimised the overall owner costs by determining the optimal time for preventive interventions in a system with minor, major and catastrophic faults, Morant et al. (2014) developed the strategies based on historical data and considering different reliability availability, maintainability and safety parameters.

2.1.2 Intervention programs

The literature on the development of intervention programs considering objects to be connected with each other within a network has mainly concentrated on the track. A first group of research has concentrated on scheduling required interventions considering resource, spatial, or temporal constraints (Budai-Balke, 2009; Higgins et al., 1999; Peng, 2011; Pouryousef et al., 2010). Higgins et al. (1999), the initial work in this field, proposed a scheduling model to reduce disturbances to train services, maintenance costs, and the amount of time a given track segment has a level of service below a specified threshold. The objective was a weighted combination of minimizing the expected interference delays due to delayed trains overlapping the maintenance schedule and due to delayed maintenance activities overlapping the train schedule, and prioritised finishing time of maintenance activities. This model was extended by the inclusion of unique projects to be included in the schedule and an exclusivity constraints preventing the combination of specific interventions (Budai-Balke, 2009; Budai et al., 2004, 2006), and the grouping of interventions within multiple segments (Pouryousef et al., 2010). Different to the former, Peng (2011) proposed a scheduling model including the scheduling of resources, such as the intervention vehicle and work team. It included a mutual exclusion constraint, preference constraints, limit constraints, rounding (duration of a job) constraints, minimum duration constraints, and project duration constraints. The overall objective was to minimize the total costs of the interventions and the production teams.

All the work presented so far concentrates more on the individual object and overall constraints, such as resources. They, however, have not focused on the dependencies between objects in respect of the network functionality. Most of the railway oriented research on the development of intervention programs with a specific focus on the dependencies between objects followed either a bottom-up approach (Caetano & Teixeira, 2014; Fecarotti & Andrews, 2017; Furuya & Madanat, 2013; Zhao et al., 2009) or a top-down approach (Den Hertog et al., 2005; Jenema, 2011; Van Zante-De Fokkert et al., 2007). With the bottom-up approach, optimal and near-optimal interventions are identified for each object separately before selecting the exact interventions considering all objects together on the network level. The consideration of the network varies from work to work. Zhao et al. (2009) considered only the different components of the track (e.g. rail, ballast, and sleepers) while Furuya & Madanat (2013) and Caetano & Teixeira (2014) considered additionally the relations between different segments. Zhao et al. (2009) and Caetano & Teixeira (2014) introduced a penalty cost based on the optimal intervention strategy to account for a shifting away from the optimal point in time for the execution of interventions. Furuya & Madanat (2013) considered economical, functional, and stochastic dependencies by incorporating economies of scale and capacity constraints into their optimisation model. The economies of scale were with respect to reduced costs due to shared setup and labour costs when interventions are executed simultaneously. The intervention costs were divided into fixed and variable costs. The capacity constraint was implemented by the introduction of capacity thresholds on routes in the network assuring that a minimum capacity is provided in all time. Fecarotti & Andrews (2017) proposed a Petri-Net based model to develop intervention programs for entire rail lines. The optimal interventions on the objects were defined using a Petri-Net simulation while the network optimisation model was modelled as a knapsack problem. The two-step approach of first determining the optimal intervention strategies and then considering network dependencies in the determination of the optimal intervention program used by all these works has both advantages and disadvantages. The advantage is that it reduces the solution space, which decreases the combinatorial problem. The disadvantage is that one is not sure that the optimal solution can be found.

In the works in which a top-down approach was used, an optimal track occupancy schedule was determined which was then used to help minimize the traffic disturbance (Den Hertog et al., 2005; Jenema, 2011; Van Zante-De Fokkert et al., 2007). Den Hertog et al. (2005) developed a methodology to divide a railway track network into optimal intervention work zones. They used rules dependent on constraints considering the time in the day when interventions are executed, the maximum length of work zones due to limited resources, and the maintenance intervals. Based on this work, Van Zante-De Fokkert et al. (2007) developed a two-step methodology to create an optimal four-week maintenance schedule of the work zones. First, single-track grids were built, which are sets of work zones that can be out of service simultaneously. Second, these grids were assigned to nights. In order to get the optimal schedule in terms of minimum disturbance and costs and in order to fulfil all constraints, e.g. a maximum workload per night, the methodology was iterative. A similar model was developed by Jenema (2011), who proposed a train-free period scheduling model based on necessary interventions. The model minimised the occupancy costs, maintenance costs and project costs. Similar to the work of Van Zante-De Fokkert et al. (2007), the infrastructure was split up into work zones, and the model considered the different object categories, e.g. track and switches.

Beside the bottom-up and top-down models, Pargar (2015) developed a different model to define the long-term optimal intervention program for an entire track network. The model considered the grouping effect over all components of a single section, the benefit from grouping multiple sections together, and the last possible time for carrying out the next intervention. The objective was to minimize the costs of projects by grouping them. The problem was solved by an integer linear program which, however, showed weaknesses when the network became bigger. Lethanh & Adey (2016) took the stance that the development of intervention programs on networks with multiple owners requires first determining when service is expected to be interrupted and then deciding what interventions are to be executed in the area. They used real options to take into consideration the large uncertainty related with future intervention need.

Rather than looking for the optimal intervention program by investigating many possible combinations, Guler (2013) used decision rules, based on interviews with railway infrastructure experts, to reduce the vast number of possible combinations. The implemented expert system defines the required interventions, from which the optimal set is selected considering resource availability constraints. Guler, however, does not test his results for optimality, especially in respect to the strategies determined based on decision rules.

Regarding bridges, Frangopol & Liu (2007) developed optimal intervention programs for multiple bridges in a connected network using the costs of intervention and a reliability measure to represent the risk on entire links. They took into consideration uncertainties related to deterioration using Monte-Carlo simulations and a genetic algorithm to determine the optimal strategy.

Chen et al. (2013) proposed three different optimization models for developing intervention programs on railway power supply systems considering reliability. A first model minimized the costs including intervention costs and penalty costs for none-optimal execution time, while introducing a minimum reliability constraint. A second model maximized the reliability consisting of the initial reliability, deterioration process and the improvement of an intervention, while the costs are limited by a constraint. The third model proposed combined the costs minimization and the reliability maximization in a two-objective model.

2.2 Other infrastructure

Since not only railway infrastructure mangers have to develop intervention programs, the development of intervention programs considering the network connectivity as well as the risk reduction of interventions can be seen in literature about other infrastructures. For example, roads (Eicher et al., 2015; Hajdin & Adey, 2006; Lethanh et al., 2018, 2014), inland water ways (Kielhauser et al., 2017), water distribution networks (Kerwin & Adey, 2017), and developing intervention programs for multiple urban networks (Kielhauser & Adey, 2017; Kielhauser et al., 2016). Hajdin & Adey (2006) introduced an integer linear program based on a network flow model to develop optimal intervention programs consisting of single work zones on a highway network considering a limited budget. This model has been further extended (Eicher et al., 2015; Lethanh et al., 2018, 2014). The extension done by Lethanh et al. (2014) allows the determination of the optimal set of work zones in a road network. In addition to the maximum work zone length constraint from the previous work, a minimum distance between two work zones constraint was included. Eicher et al. (2015) developed an algorithm to automatically set up the required constraint matrix of the optimization problem, which allowed to apply the model on a large scale network within a GIS framework in Lethanh et al. (2018).

3 Background

This chapter provides an overview of railway infrastructure networks and its maintenance to ensure that it is clear which aspects are considered in the proposed methodology. The first section (3.1) identifies the different infrastructure objects within the railway infrastructure. The second section (3.2) looks at the railway infrastructure from the network point of view and identifies the different topological elements. The third section (3.3) concentrates on the possible interventions and their characteristics. The fourth section (3.4) identifies the different dependencies between objects within a railway infrastructure network. The last section (3.5) discusses the importance and effects of time windows in respect with the execution of interventions.

3.1 Infrastructure objects

A railway infrastructure network consists of thousands of objects of different categories. Table 1 shows the classification used in this work, which is adapted from a categorisation in the directive 2012/34/EU from the European Union (European Union, 2012). More general, object categories are further divided into sub-categories, where the sub-categories have different influences on the railway network. For example, switches and crossings are similar sub-categories in respect to their construction and material. They, however, differ widely in their functionality. While a crossing only allows one railway track to cross another one, a switch enables a train to change tracks. The differentiation of switch and crossings in sub-categories is important in the analysis of the functionality of the railway network. In the proposed methodology, components are not considered because the consideration of objects on the level of categories, or where possible, on the level of sub-categories, is accurate enough for developing intervention programs. This means that even though a track consists of rails, sleepers, fastenings, and ballast, it is considered as one single object.

Table 1. Object categories in a railway infrastructure network - adapted from (European Union, 2012)

Object categories	Sub-categories	Type of category	Description
Track		Continuous	Superstructure on which trains run; Consist of rails, sleepers, fastenings, ballast, sub-ballast, formation layers and subsoil.
Earthwork	Embankments	Stand-alone	Required to enable the track routing; May consist of geotextiles, drainage channels and trenches, culverts, planting for protecting side slopes, etc.
	Cuttings		
Switches and crossing	Switch	Stand-alone	Track configuration elements
	Crossing		
Engineering structures	Bridge	Stand-alone	Serve the purpose of conflict-free crossing, overcoming topographical obstacles, and protecting against natural hazards.
	Tunnel		
	Retaining wall		
	Protection structures		
Level crossing		Stand-alone	Allow crossing of railway and road on the same level; Include appliances to ensure the safety of road traffic
Access ways	Passenger platforms	Stand-alone	
	Good platforms		
Safety, signalling and telecommunications installations	Safety installations	Dispersed	Ensure safety, controlling, and communication. Consist of fixed signals, track circuits, train control equipment, signal cables or wires, signal boxes, control systems, telecommunication network for tunnels, radio, etc.
	Signalling installations		
	Telecommunications installations		
Electric traction power supply	Production plant	Stand-alone	Power supply network consisting of all elements between the production and the catenary
	Substations	Stand-alone	
	Supply cables	Stand-alone	
	Catenaries	Continuous	
Lighting installations		Dispersed	Lighting installations for traffic and safety purposes
Buildings		Stand-alone	Used by the infrastructure department

The column *Type of* category in Table 1 characterises the object category according to its structure into continuous, dispersed and stand-alone object types. Continuous objects are physically connected with each other and can be seen as long one-dimensional objects (e.g. track, catenary). Even though such objects are divided into single objects following mostly a homogenous segmentation, they do not have an obvious structural separation. For example, track is usually divided into 200-meter long sections for monitoring reasons, but it is impossible to say with naked eye where a track section stops and another starts. Stand-alone and dispersed objects can be easily identified due to the structural or functional independencies between objects. Stand-alone objects are single objects at a single well-defined location within the network, e.g. a bridge, a tunnel. Dispersed objects are objects that consist of multiple components at multiple well-defined locations within the network. For example, a particular signalling installation consists of different components, such as fix signals, cables, and train detection components, that are located at different locations along the track. All these components are needed for this particular signalling installation to function.

The classification of an object as continuous, stand-alone, or dispersed is not always simple. For example, a viaduct can be seen as a continuous bridge section, one long bridge or as many small adjacent bridges. A cutting, which is an earthwork, can be seen as a continuous object, one single large object or as many small objects. In the proposed methodology, these are, however, both considered as stand-alone objects. In the case of the bridges, this is because they can be divided into clear distinct objects (e.g. a viaduct in multiple parts). In the case of the cuttings, this is because they are independent of the complex track topology (e.g., a long spread cutting follows the line routing and not the track topology).

3.2 Topological network breakdown

Network topology must be considered when planning interventions in order to appropriately take into consideration losses of services due to the execution of interventions. For this methodology, a network is considered to be composed of lines, which consist of the entire infrastructure connecting two stations (Figure 1). Each line is divided into routes, which build the track routes between two switches where trains are able to switch routes. Routes are further divided into sections being characterized by their homogenous cross sections over their length or the existence of one or more switches within a close area. Each infrastructure object is assigned to the routes of which they are part.

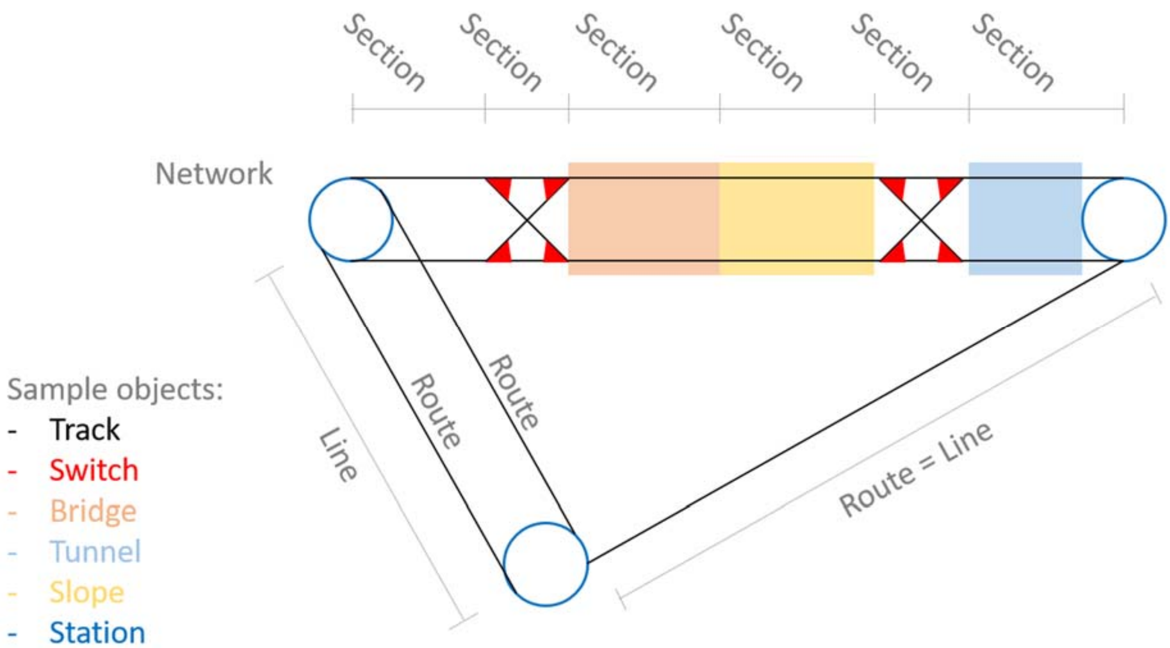

Figure 1. Railway network breakdown

3.3 Interventions

The state of an object can be improved by executing interventions. The possible interventions depend on the object category and state. In order to stay on the focus of this work, a detailed description of possible interventions for each object category is omitted. Instead it is referred to the existing literature of reports from infrastructure organisations (SBB Infrastruktur, 2016, 2017), projects deliverables (Aksentijevic et al., 2015; SMARTRAIL, 2014a, 2014b), and technical literature (Esveld, 2014; Fendrich & Fengler, 2013; Freystein et al., 2015; Gutsche, 2009; Jänsch, 2016; Kiessling et al., 2009; Mehlhorn, 2010; Profillidis, 2014). In the presented work, the interventions considered are, independently of the object category, either 1) minor rehabilitation interventions that improve the state slightly, 2) major rehabilitation interventions that improve the state significantly but do not restore the object to a like new state, and 3) renewal interventions, which bring the object back to a like new state. Even though, deterioration is the major driving factor for executing interventions, it is neglected in this work. In a first stage, the work concentrates on developing optimal intervention programs for one single year. Nevertheless, if deterioration is considered, it is important to consider different deterioration rates for different object categories. For example, a switch deteriorates much faster, and therefore, requires interventions much more frequently, than a bridge. This increases the complexity because some interventions, e.g. grinding of switches, are executed every 1 to 3 years, which would mean that these interventions are included in every second year of an intervention program with yearly time steps. On the other hand, a bridge only requires an intervention on average once in every twenty years.

Interventions have two main characteristics, they cost money and require a certain length of time to be executed. For some interventions, the costs can be divided into variable and fix costs, where the latter is independent of the size of the intervention. Service interruption due to the execution of an intervention also has characteristics. The two most important are the track occupancy and the traffic impact of an intervention. The track occupancy describes the requirement of going on the track for executing an intervention, while the traffic impact describes whether the traffic operation has to be stopped during the execution of the intervention. Most interventions with track occupancy require a closure of the track for operation, but not all. Some interventions can be executed without interrupting service. For example, minor interventions on the fastening system can be executed within a short track occupancy between two running trains.

In the presented methodology, all interventions are classified into five types (Table 2). Continuous interventions (type 1) are executed continuously following the track topology using intervention trains. Due to the usage of such intervention trains, which run along the track, the track cannot be used for any other functionality, i.e. track occupancy is required, and the route has to be closed to traffic. Major interventions within the track clearance (type 2) require track occupancy due to work executed within the required clearance area, machinery using the track, or being required to deconstruct the entire track, e.g. bridge rebuild. Independent of the source of track occupancy, the traffic route has to be closed for operation. The traffic route may also need to be closed without the intervention requiring track occupancy. This may be because of the execution of off-track interventions where the power supply needs to be cut or safety equipment need to use the track. Interventions on dispersed objects may also lead to a route closure to traffic. These interventions are classified as major track disabling interventions (type 3). Regarding interventions on dispersed objects, it is assumed that even though these interventions may be executed within the track clearance, they can be arranged around other track possessing interventions. Intervention type 4 refers to minor on-track interventions that can be executed without interrupting operation. With the implementation of a speed reduction on the affected route, and the installation of a train approaching alarm system, these interventions can be executed in between the passing of trains. The final type of intervention (type 5) includes all interventions that are off-track and do not have an impact on the traffic operation.

Table 2. Intervention types for railway infrastructures

Intervention type		Description	Track occupancy of intervention	Traffic impact	Examples
1	Continuous intervention	Interventions executed continuously along the track	Track is occupied due to intervention trains	Route closure	Interventions on track and catenary
2	Major on track intervention	Interventions within the track clearance	Track is occupied due to either work in the track field or the deconstruction of the track during the execution	Route closure	Major interventions on engineering structures, earthworks, level crossings, and switches
3	Major track disabling intervention	Interventions within the track clearance of dispersed objects or traffic affecting interventions on off-track objects	Track is not occupied by the intervention	Route closure	Interventions on interlocking blocks; Interlock rehabilitations; Power sub-station rehabilitations
4	Minor on-track intervention	Interventions within the track clearance under operation	Only short time track occupancy (in between of operating trains)	Speed restriction	Minor switch interventions
5	Off-track intervention	Interventions outside the track clearance	No track occupancy	None	Minor rehabilitations on engineering structures and earthworks

3.4 Network dependencies

In a railway infrastructure network, there are dependencies between objects. These dependencies change how stakeholders are affected, e.g. intervention costs or traffic disturbance costs, when interventions are grouped together. It is assumed that grouped interventions are either executed parallel or sequential in time.

Concerning the effect of combining interventions, especially interventions on objects from different object categories and the possibilities of combining such interventions, the railway specific literature does not provide any information. In other fields, however, the consideration of economical, functional/structural, and stochastic dependencies within a multi-component system is extensive (Dekker et al., 1997; Furuya & Madanat, 2013; Van Horenbeek & Pintelon, 2013). Economical dependencies refer to the possible difference between the costs of executing multiple interventions together when compared to the summed costs of individual execution of the same interventions (e.g. economies of scale, shared fix costs). Functional, or sometimes structural, dependencies refer to situations where one objects functionality depends on another objects functionality. Stochastic dependencies refer to situations where the probabilities of object failures are correlated, e.g. due to common cause failures. Olde Keizer et al. (2017), however, have stated the necessity of considering more types of dependencies when analysing systems. They included resource dependencies (e.g.

material, budget) and divided functional dependencies into technical and performance oriented dependencies. Technical dependencies refer to the relation between objects, with respect to their functionality as an object. This refers to the description provided above for the functional dependencies. The performance-oriented dependencies introduced by them, refer to the relation between the objects and the overall performance of the system. They refer to the structural connectivity of the system (e.g. serial, parallel, k-out-of-N, redundancy).

Only three out of the five types of dependencies listed above are identified as important for planning interventions on a railway infrastructure network: technical, which are from now on referred as structural dependencies, economical, and performance-oriented dependencies, which are from now on referred as topological dependencies. These three types of dependencies in railway infrastructure networks are illustrated graphically in Figure 2 on an exemplary double track line consisting of eight track objects, four switches, and four bridges. On top, the entire infrastructure network is shown, which is then divided into three object categories. The network topology at the bottom represents the routes used by traffic to operate. The arrows (red and grey) and the blue groups indicate the different dependencies. They are clarified in more detail in the following subsections.

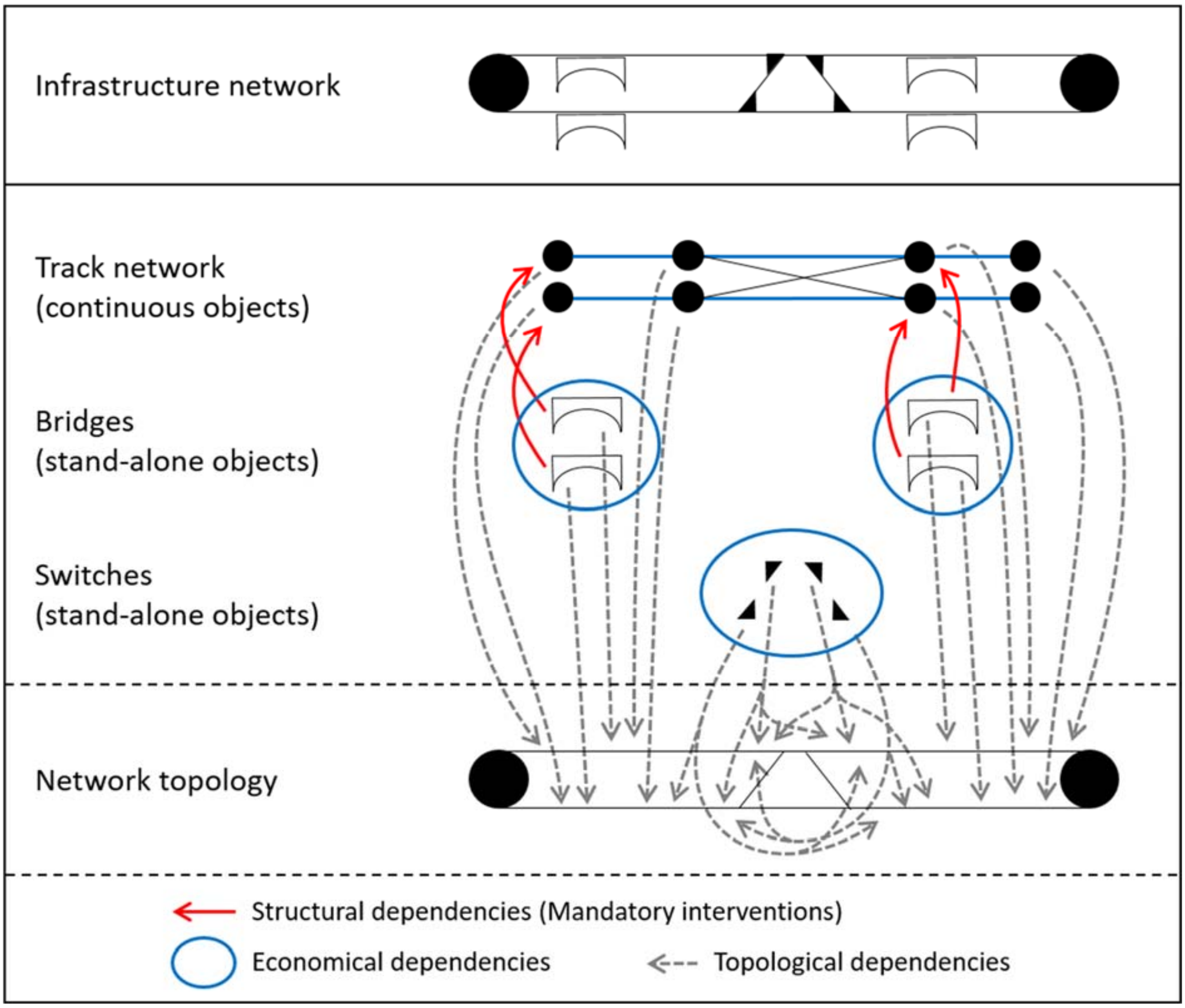

Figure 2. Network dependencies

3.4.1 Structural dependencies

Objects may be structural dependent on other objects (red arrows in Figure 2), meaning that an intervention on one object leads to a mandatory intervention on the other. For example, a rebuild of a bridge entails the rebuild of the carrying track infrastructure.

3.4.2 Economical dependencies

Object A is economically dependent on object B due to interventions I_A performed on object A and I_B performed on object B, when the costs of a grouped execution C_{IA+IB} is less (or more) expensive than the sum of the individual costs $C_{IA} + C_{IB}$ (blue circles in Figure 2). For example, the costs for interventions on two different switches that lie close to each other are less if they are executed within the same shift than if they are executed at two different moments in time. In order to be economical dependant, two criteria have to be fulfilled, 1) the objects must be within a relevant distance, and 2) the interventions on the objects need to have common fixed costs. The relevant distance depends on the intervention type and the object category. For continuous objects, objects are close when they are physically connected within the track topology, so that the intervention trains can work continuously from one object to the next. For stand-alone and dispersed objects, closeness depends on the ability of the objects to be grouped into groups, where the execution of interventions on multiple objects would reduce the total intervention costs when compared to executing them separately. In the example shown in Figure 2, the bridges parallel to each other are close to each other, while the two pairs of bridges are too far from each other to be economical dependant.

3.4.3 Topological dependencies

If during the execution of intervention I_B on object B, object A cannot provide an adequate level of service, then it is considered that object A is topologically dependent on object B due to intervention I_B (grey dashed arrows in Figure 2). In general, the total loss in level of service due to the execution of the intervention on one object can be either reduced or increased when multiple interventions are executed at the same time. On one hand, the parallel execution of two interventions within the same route reduces the overall loss in level of service compared with individual executions because the total duration can be limited to the duration of the longer of the two interventions. On the other hand, the execution of two interventions on parallel objects may increase the losses in level of service when executed together. This is because the parallel layout of the objects may lead to a complete closure of the line and, therefore, to much higher losses than if the interventions were executed separately.

Considering the different intervention types identified in the intervention section above, three different combinations of topological dependencies are identified.

- A continuous intervention (intervention type 1) is executed along the track and requires the track occupancy over the entire route. It is, therefore, not possible to execute another track occupying intervention, neither another type 1 intervention nor a type 2 intervention, on the same route at the same time. For example, when a track is renewed along a route, no major bridge intervention requiring track occupancy can be executed at the same time.
- Multiple major on-track interventions (intervention type 2) can be executed at the same time on the same route. These interventions are local but lead to the closure of an entire route. Another local intervention can, therefore, be executed within the same route without any

additional loss in level of service. For example, major interventions on two bridges that are along the same route can be executed at the same time since they do not interfere with each other.

- Major track disabling interventions (intervention type 3) are interventions that lead to a loss in level of service without requiring track occupancy. Such an intervention can be executed at the same time as any track occupancy intervention (intervention types 1 and 2) or another major track disabling intervention (intervention type 3), and, therefore, share the losses in level of service, as long as both interventions affect the same routes. For example, a track intervention can be executed while an intervention on the power supply system is executed that requires to turn off the electricity, but does not require track occupancy.

3.5 Time window of intervention execution

An important element of executing interventions on railway infrastructure are the time windows used to do so. Time windows, sometimes also referred to as intervals, describe the time of the week over which an intervention is executed, e.g. weekday, weekend, night. The time window of execution influences the traffic volume that is affect by an intervention, the costs of an intervention, its duration, and the potential cost reduction (or increase) due to economical dependencies.

Most common are intervention time windows during the night when little to no traffic runs on the network. During such a time window, the loss in level of service is much smaller than during the peak hours' time. The determination of possible time windows depends on the traffic volume and density variability on the network. Their length can vary between different lines, as different lines have different traffic volume variability during the day, e.g. the traffic on a main line can be very different from the traffic on a siding line.

While executing interventions during the night might reduce the losses in level of service by taking advantage of the lower traffic volume during this time, it may have higher expenses than executing interventions during the day due to additional requirements, for example night supplements for the workers and additional equipment.

Further, the time window influences the duration of an intervention. It has to be long enough so that the intervention can be executed within it. An exception is when the execution of an intervention can be divided into multiple time windows. For example, a continuous track intervention can be stopped anywhere and resumed at this location in the next time window. This division, however, usually increases the total duration of the execution of the intervention. The duration of execution consists of the three elements 1) the time to set-up of the intervention (e.g. the crew has to go out, the equipment has to be positioned), 2) the effective work time, and 3) the clearing time after the execution of the intervention (e.g. removing all equipment, withdrawal of the crew). The set-up and clearing time together are referred to herein as lost time. The division of an intervention increases the total duration because the lost time has to be added for each window used. Figure 3 illustrates this effect by dividing a continuous executed intervention into three night windows. Each night window requires its separate setup and clearing time (black part in Figure 3). It should be noted that a portion of the intervention costs may be increased for a continuous execution due to the need of more than one crew to cover the duration of the continuous intervention.

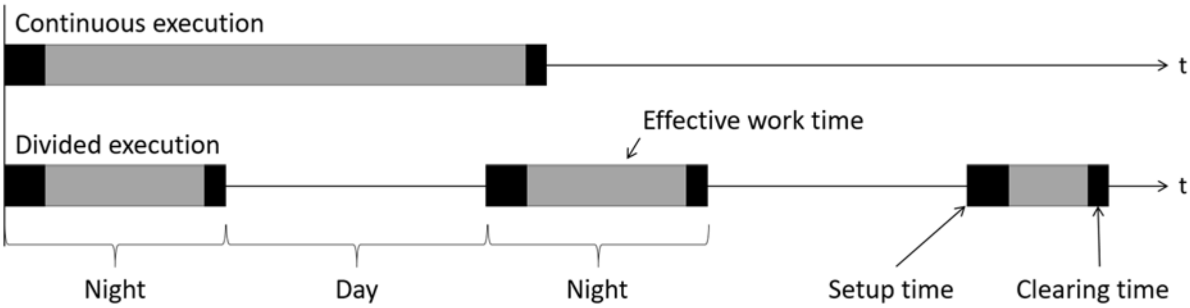

Figure 3. Effect of time windows

The lost time has two effects on the intervention program. First, each time window usually has a shorter effective work time in which interventions can be executed compared with their maximal time window length. Second, the duration of the loss in level of service is, usually, longer than the effective intervention duration. The duration of the loss in level of service is equal to the effective intervention duration plus the sum of all lost time. For example, an intervention takes 20 hours to be executed and has 2 hours of lost time due to setting it up and clearing afterwards. This intervention could be executed either during a weekend window with a maximum duration of 50 hours and a maximum effective work time of 48 hours (50h – 2h), or by being divided during multiple night windows with a maximum duration of 6 hours and a maximum effective work time of 4 hours (6h – 4h). While the intervention can be executed during one weekend window, it would require 5 night windows to do the same (20h / 4h per night). The total duration of loss in level of service would equal to 22 hours (20h + 2h) when executed during a weekend window, and to 30 hours (20h + 5*2h) when executed during night windows. The duration for a night execution is much longer than the weekend execution due to the additional lost times for each new window used. The decision for a weekend or a night execution would now depend on the costs related to the duration and the traffic volume during both time windows.

4 Conceptual model

4.1 Objective function

4.1.1 General

As stated earlier, an intervention program is a list of interventions to be executed on objects within a specified time period taking into consideration dependencies. It states in which type of time window (e.g. during a weekday, weekend, or night) the interventions are to be executed and specifies the groups of interventions that are executed together (parallel or sequential in time). The optimal intervention program is defined as the one that maximises the net benefit considering the benefit in terms of the reduction in risk and the costs for the owner and user due to the execution of interventions (equation 1).

$$Max\ Z = Benefit - Costs = (R_0 - R_{IP}) - (C_{Owner,IP} + C_{User,IP}) \quad 1$$

4.1.2 Benefits

The risk reduction is defined as the difference between the risks related to the state of the objects when no intervention is executed (R_0) and the risk related to the object states improved by the interventions (R_{IP}). The risk is estimated based on the risk assessment process developed in task 3.2 of the Destination Rail project (Papathanasiou et al., 2016). There, a process is developed that can be used to estimate risks on different levels, e.g. object, section, route, and network. On the object level each object is considered individually. On the other levels, objects are grouped together and the risk is estimated based on all objects within their groups. The other levels allow the consideration of dependencies between different objects when estimating risk. For example, the failure of a track may be more likely when the underlying bridge has already failed, keeping in mind here that failure only means that it does not provide an adequate level of service and not necessarily collapse. The risk is estimated based on different risk scenarios, where each scenario describes one specific chain of events happening. Each event happens with a certain probability, and the probability of the scenario is the product of all events of this scenario. The multiplication of the probability of a scenario with the consequences arising when this scenario occurs yield the risk related to this particular scenario. The sum of the risk related to all possible scenarios gives the overall risk related to the system. Equation 2 provides the formula, where R is the overall risk, $Cons_j$ the consequences of scenario j, and $Prob_{e,j}$ the probability of event e in scenario j. While the consequences of a scenario remain the same for both risks, R_0 and R_{IP}, the probability depend on the state of the objects, which is affected by the execution of interventions.

$$R = \sum_{j=1}^{J} (Cons_j * Prob_j) = \sum_{j=1}^{J} \left(Cons_j * \prod_{e=1}^{E} Prob_{e,j} \right) \quad 2$$

4.1.3 Costs

The costs refer to all costs incurred due to the execution of all the interventions in the intervention program. This includes the costs related to the interventions incurred by the owner ($C_{Owner,IP}$) and by the users due to losses in the level of service while executing the interventions ($C_{User,IP}$).

The owner costs refer to all intervention related costs carried by the owner. They are divided into variable and fix cost, wherefrom fix costs can be shared by different interventions. Equation 3 states this general form of the owner costs by summing up the fix costs of each group of interventions g ($c_{fix,g}$), where a group of interventions can consist of only one intervention, and the variable costs of each intervention i ($c_{variable,i}$).

$$C_{Owner,IP} = \sum_{g=1}^{G} c_{fix,g} + \sum_{i=1}^{I} c_{variable,i} \qquad 3$$

The user costs refer to all costs incurred by the user because of the execution of interventions executed. When executing interventions traffic can be interrupted in different ways, which is modelled here as in different traffic states. A traffic state is defined by the combination of a specific track configuration considering certain route closures, and the time window of execution. For example, traffic state A refers to the closure of route 1 and 2 during daytime. The user costs are the sum of the costs related to all traffic states, where the cost related to each traffic state is the product of the losses in level of service per time unit due to this traffic state ($LLOS_{ts}$), the total duration of this traffic state ($Duration_{ts}$), and the value of time (VOT). This general formulation of the user costs is shown in equation 4.

$$C_{User,IP} = \sum_{ts=1}^{TS} (LLOS_{ts} * Duration_{ts} * VOT) \qquad 4$$

Figure 4 provides an overview of how the different characteristics of a railway infrastructure network influence the costs of an intervention program. Therefore, the owner and user costs are divided into their elements according to equations 3 and 4 first. Second, all factors influencing this cost elements are listed in the lowest row. These factors are related to characteristics of the interventions (dark blue), the network characteristic of the infrastructure (light blue), the time window of the execution (violet), and the value of time (orange).

Owner cost			User cost						
Individual costs	Shared costs		Loss in level of service			Duration			Value of time
Variable intervention costs	Fix intervention costs	Economical dependencies	Traffic restriction	Network topology	Traffic volume	Intervention duration	Topological dependencies	Lost time	Value of time

Legend: Interventions | Infrastructure Network | Time window | Value of time

Figure 4. Cost of an intervention program

The owner costs consist of individual costs and shared costs as it can be seen in equation 3. The individual costs are only influenced by the variable intervention costs, while the shared costs depend on the fixed intervention costs and the economical dependencies between interventions on objects. Economical dependencies exist only when the same intervention is executed on objects close to each other. The definition of closeness varies as a function of the intervention type. Continuous interventions are close when they are executed on topological connected objects, while stand-alone interventions are close when they are within a particular range allowing the crew to use more or less the same set-up.

The user costs sum up the costs for each traffic state (equation 4). The cost estimation of a traffic state consists of the loss in level of service per time unit, the duration, and the value of time. The loss in level of service refers to the losses occurred per time unit due to the traffic restrictions required by the interventions, the network topology, and the traffic volume affected within the time window of the execution. The duration refers to the length in time in which this traffic state with a loss in service is used. It depends on the intervention durations, the topological dependencies allowing the total duration to be reduced, and the lost time of set-up and clearing dependent on the traffic window of the intervention execution. Regarding the colouring of the lowest level in Figure 4 it becomes clear that both, the loss in service and the duration depend on the intervention selected, the network topology, and the time window of the execution. Further, the user costs depend on the value of time, which is required in order to monetarise the impact of the loss of service.

The costs of an intervention program depend strongly on the exact times of execution of interventions within a time period. For example, if two interventions are to be executed within one year and each only take one week, it is important to know if they are to be executed at the same time, in two consecutive weeks, or two totally different weeks. Interventions need to have overlapping execution times to take advantage of the topological dependencies, while the advantage of economical dependencies require a consecutive execution. Secondly, the time window in which an intervention is executed has a strong influence in the costs. Regarding Figure 4, both, the losses in service and the duration depend on traffic window characteristics, which usually influence the costs in the opposite way. For example, an intervention can be executed during a weekend time window or during multiple night windows, where the traffic volume during the night is smaller, but the duration is increased due to more lost time. These considerations of time within each time period increases the complexity of the optimisation.

4.2 Constraint types

Not all intervention programs are possible. Their development is subject to constraints. These are either organisational or structural in nature. Organisational constraints (equation 5) are for example resource limitations that may make an intervention program with a high net benefit unfeasible. Equation 5 shows the most general form of an organisational constraint, where the sum of all k_i has to be smaller than a maximal value Ω_{max}. Structural constraints (equation 6) arise from structural dependencies defining mandatory intervention combinations due to the structural construction of the objects. Equation 6 shows an exemplary formulation of a structural constraint where element 2 has to be selected if element 1 is selected. Thereby, δ_i is a binary variable that is 1 if element i is selected and 0 otherwise.

$$\sum_{i=1}^{K} k_i \leq \Omega_{max} \qquad 5$$

$$\delta_1 - \delta_2 \leq 0 \qquad 6$$

4.3 Structure

4.3.1 General

One way of solving the general optimisation model presented in the former section is to model the system as a constrained network flow model. In general, a network flow model represents a certain system with nodes and edges connecting nodes and guiding certain flow from the source nodes to the sink nodes along the edges of the network (Bertsekas, 1998; Jungnickel, 2013; Subramanian et al., 2016). It is only constrained by the conservation of flow along the network and the flow capacity on the edges. Different types of flow models are used to solve different problems, e.g. assignment problem, shortest path, maximum flow problem, or minimum cost flow problem. Extensions of network flow models are required to solve special problems. Three ways that a network flow model can be extended are 1) introduce different types of flow to be able to model different flows within one network, i.e. different means of transportation in a traffic model 2) introduce gains and losses that release the flow conservation constraints and enable different system behaviours to be modelled, i.e. leaking pipes in a water network, and 3) introduce other constraints, i.e. maximum path lengths of postmen in a routing problem.

The network flow model used in this work to develop intervention programs includes extensions of the three types mentioned. The network flow model can be seen as a constrained multilayer minimum cost flow problem. The network is built as a multilayer network consisting of nodes, inlayer edges and interlayer edges. Inlayer edges describe the node connections within one layer while the interlayer edges connect nodes from one layer to a node in another layer. Figure 5 illustrates the concept behind the network model using a small example where there are 7 possible interventions that can be executed in in one of four possible traffic states. The black numbers in brackets are the edge costs. The blue numbers show the durations of the interventions. The red numbers represent the flow values of an exemplary intervention program.

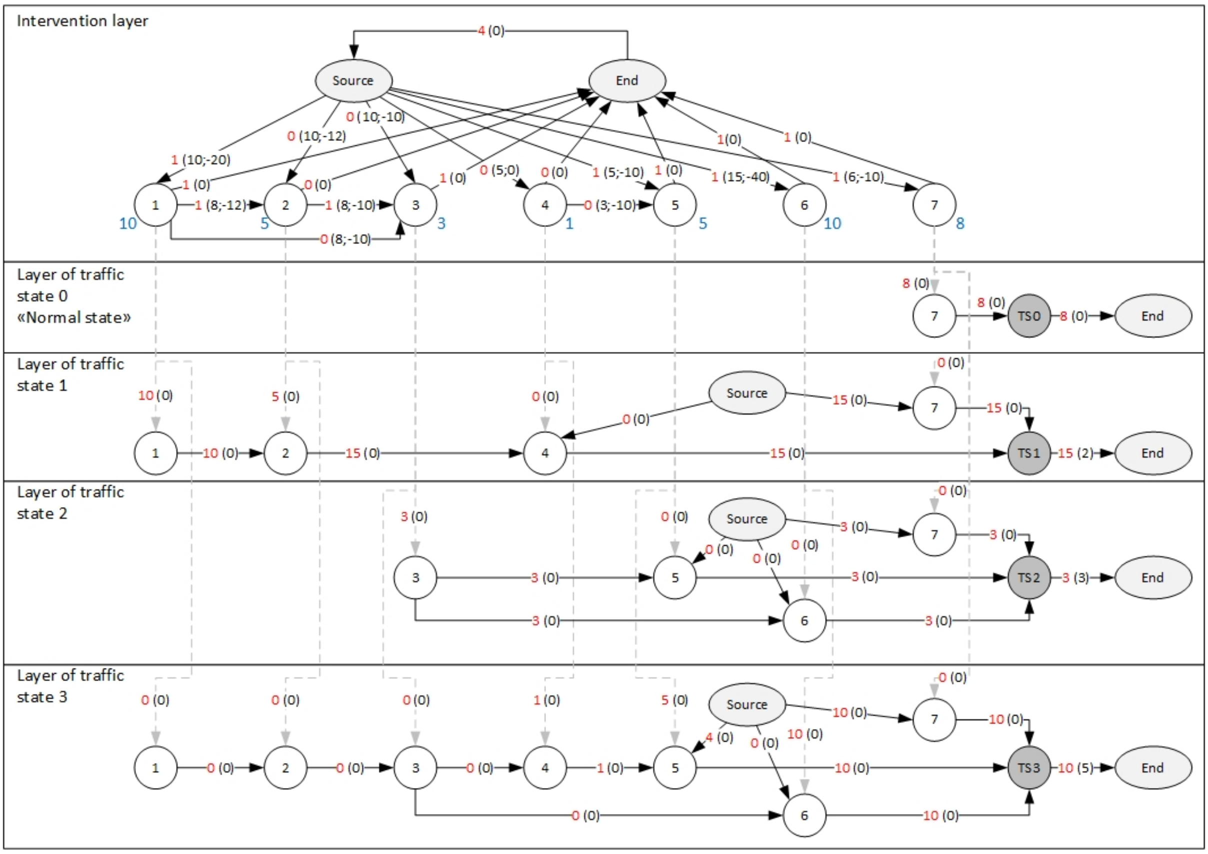

Figure 5. Simplified illustration of the network flow model

The network consists of an intervention layer and multiple traffic state layers. The intervention layer contains information pertaining to the seven interventions to be executed. The traffic state layers contain information pertaining to the traffic states required when the interventions are being executed. Traffic state 0 refers to the network being fully operational. The location of intervention 7 in this traffic state means that it can be executed without interrupting traffic. Interventions 1, 2, 4 and 7 can be executed when the network is in traffic state 1. Interventions 3, 5, 6 and 7 can be executed when the network is in traffic state 2. All interventions can be executed when the network is in traffic state 3.

The following two sections 4.3.2 and 4.3.3 describe the network elements of nodes and edges, before sections 4.3.4, 4.3.5, 4.3.6 and 4.3.7 explain the intervention layer, the relationship between the intervention layer and the traffic state layers, the traffic state layers, and the side constraints captured in the network model more detailed.

4.3.2 Node types

The set of nodes are divided into source nodes, end nodes, intervention nodes, and traffic state nodes. The intervention layer consists of a source and an end node generating and absorbing flow in the network. The intervention nodes in the intervention layer represent the set of possible interventions on objects. Each node represents a specific intervention on an object executed within a given time window. Each traffic state layer consists of the intervention nodes of interventions that can be executed under the specific traffic state, a traffic state node representing the traffic state itself, and an end node that absorbs the flow within the network. A traffic state layer can contain a source node that provides the network with additional flow used to equalise the flow on parallel executed interventions. The description of the node types are summarised in Table 3.

Table 3. Node types per layer

Node type	Intervention layer		Traffic state layer	
	Description	Example	Description	Example
Source	used to indicate the number of intervention groups that exist (determined considering the economical dependencies).	A value of four attributed to the source node means that four intervention groups exist, where some costs for the owner can be shared.	is used to increase the flow of each non-maximal branch to the flow of the maximal branch.	Traffic state 3: Flows of 4 and 10 from the source node to intervention 5 and 7, are required because the outgoing flows of intervention 5 and 7 would be 6 and 0, respectively, which is smaller than the outflow of 10 at intervention 6.
End	used to ensure that the number of economical intervention groups fed into the network are taken out of the network.	A value of four means that the four intervention groups allowed to take place are included in the intervention program.	used to record the total duration the network is being used in the way designated by the traffic state.	The network is being used in the way designated by traffic state 3 for 10 units of time.
Intervention	used to represent the possible interventions to be executed in the. There is only one possible intervention per object.	Seven interventions on seven objects are possible	used to indicate that it can be executed with the specified traffic state.	Intervention 1 can be executed when traffic state 1 exists or traffic state 3, but not traffic state 2.
Traffic state	N/A	N/A	used to determine the total duration required to have the specified traffic state by dividing the sum of all incoming flows by the number of incoming edges. The flow on all incoming edges has to be equal	The flow on the outgoing edge of *TS3* is 10, which is equal to the flows on the edges coming from interventions 5, 6 and 7, respectively.

4.3.3 Edge types

The edges are divided into the set of inlayer edges, where edge $e^{L}_{u,v}$ is the edge between the node u and v in layer L, and the set of interlayer edges, where edge $e^{L\text{-}K}_{v}$ is the edge between node v in layer L and its counterpart in layer K. The inlayer edges connect different nodes within the same layer. The interlayer edges connect the same intervention nodes across different layers, but do not connect different intervention nodes across different layers. The descriptions of the edge types are summarised in Table 4 and Table 5 including whether the edge is considered to be a binary edge, a non-negative integer edge or a non-negatice continuous edge, and how the cost of each edge is calculated. The cost of each edge is the sum of two parts, 1) a non-negative part for costs, a non-positive part for benefit. The net benefit of the intervention program is estimated by multiplying the values of the flow variables associated with each of the edges (either binary or non-negative real number) and the costs associated with each of the edges. An edge with a zero cost keeps the network flow rules intact but does not contribute to the cost or benefit of the intervention program when selected.

Table 4. Edge types (part 1)

Edge type	Edge	Description Edge between…	Example	Type	Costs
Inlayer in intervention layer	$e^I_{S,v}$	… the source node *S* and intervention node *v* - Used to represent the selection of an intervention and both the fixed and variable costs of the intervention are counted.	A value of 1 on the edge between the source and intervention node 1 means that intervention 1 is selected and the fix and variable costs of intervention 1 (10 monetary units) are counted.	Binary	Fix costs and variable costs of the intervention related to *v* ($C_{fix,v}$ + $C_{var,v}$) minus the risk reduction that results from the execution of intervention *v* ($R_{0,v} – R_{ip,v}$)
Inlayer in intervention layer	$e^I_{u,v}$	… intervention nodes *u* and *v* - Used to represent the selection of an intervention and only the variable costs of the intervention are counted.	A value of 1 on the edge between nodes 1 and 2 means that intervention 2 is selected and shares fix costs with intervention 1	Binary	Variable costs of the intervention related to *v* ($C_{var,v}$) minus the risk reduction that results from the execution of intervention *v* ($R_{0,v} – R_{ip,v}$)
Inlayer in intervention layer	$e^I_{v,E}$	… intervention node *v* and the end node *E*. Used to indicate that a group of interventions has ended.	A value of 1 on the edge between intervention node 3 to the end node means that intervention 3 is the last intervention of an intervention group	Binary	0
Inlayer in intervention layer	$e^I_{E,S}$	… between the end *E* and the source *S* node - Used to ensure flow conservation in the mathematical model.	A value of 4 means that the fixed costs of executing interventions were incurred four times, i.e. there are four groups of interventions	Non-negative integer	0
Inter-layer between the intervention layer and the traffic state layer	e^{I-TS}_v	… between the intervention node *v* in the intervention layer and its counterpart in the traffic layer *TS* - Used to transform the binary intervention selection from the intervention layer to the intervention duration.	A value of 5 on the edge between intervention node 2 in the intervention layer and intervention node 2 in the traffic state 1 layer means that intervention 2 is executed when the network is being used as defined with traffic state 1 with a duration of 5 time units	Non-negative continuous	0

Table 5. Edge types (part 2)

Edge type	Edge	Description Edge between…	Example	Type	Costs
Inlayer in traffic state layer	$e^{TS}_{u,v}$	…between the intervention nodes *u* and *v* - Used to sum up the duration of none-parallel executed interventions	Traffic state layer 1: A value of 10 between intervention node 1 and 2 means that intervention 1 requires 10 units of time. A value of 15 between intervention node 2 and 4 means that intervention 1 and 2 require 15 units of time.	Non-negative continuous	0
Inlayer in traffic state layer	$e^{TS}_{v,TS}$	…edge between the intervention node *v* and the traffic state node *TS* - Used to pass the required time to have the traffic state due to none-parallel executed interventions	Traffic state layer 3: A value of 10 on the edge between intervention node 5 and TS3 means that traffic state 3 is required for less than or equal to 10 units of time to execute interventions 1, 2, 3, 4 and 5, if they are executed with traffic state 3 in place.	Non-negative continuous	0
Inlayer in traffic state layer	$e^{TS}_{TS,E}$	… edge between the traffic state node *TS* and the end node - Used to convey the total time that the specified traffic state is required, which is used to estimate the user costs incurred while the network is in this traffic state.	Traffic state 3: A value of 10 on the edge between TS3 and the end node means that the network will be in traffic state 3 for 10 units of time while the interventions, which are to have this traffic state are being executed. This number is multiplied by 5, which is the user costs per unit of traffic state 3.	Non-negative continuous	User costs per unit of time due to traffic state *TS* (C_{TS})
Inlayer in traffic state layer	$e^{TS}_{S,v}$	… edge between a source node and intervention node *v* - used to equalising flow differences from split flows.	Traffic state 3: A value of 4 on the edge between the source and intervention 5 means that the summed duration of interventions 1, 2, 3, 4 and 5, if they are executed with traffic state 3, is 4 units of time less than the duration of interventions 1, 2, 3, and 6. The latter is the maximum duration of non-parallel interventions, and therefore, is the value on the edge between the source and intervention 6 equal to 0.	Non-negative continuous	0

4.3.4 Intervention layer

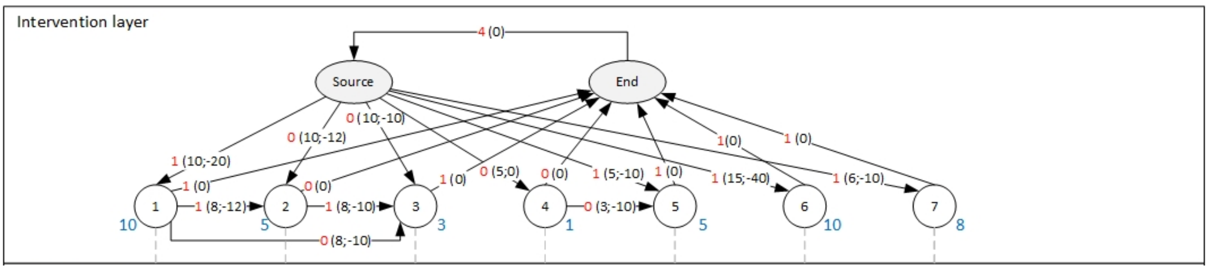

Figure 6. Illustration of the intervention layer

The intervention layer, shown in Figure 6, contains information pertaining to the possible interventions, the selected interventions, the costs and benefits directly related to the interventions and the economical dependencies when interventions are grouped together. Each intervention node v is connected to the source node (edges $e^{I}_{S,v}$) and the end node (edges $e^{I}_{v,E}$). The costs of each edge $e^{I}_{S,v}$ connecting the source with the intervention node v are the fixed and the variable intervention costs of the intervention related to v minus the benefit of reducing the risk related to the object associated to v. In Figure 6, they are written in brackets, where the first value refers to the intervention costs and the second value refers to the benefit. For example, the costs of the edge between the source and intervention 1 is labelled as (10;-20), where the costs for intervention 1 (fix + variable costs) are 10 monetary units and the benefit of this intervention is 20 monetary units. The total costs of this edge is then -10 monetary units. The costs of each edge $e^{I}_{v,E}$ connecting the intervention node v and the end node are 0 as they do not have any costs or benefits related to them and are only used to ensure the mathematical consistency in the mathematical model.

The edges between the intervention nodes u and v ($e^{I}_{u,v}$) are used to represent the execution of multiple interventions without incurring fixed intervention costs multiple times, i.e. they capture the economical dependency of two interventions. The costs of each of these intervention to intervention edges ($e^{I}_{u,v}$) are the variable intervention costs of intervention v minus the risk reduction related to intervention v.

The network within this layer is constructed as a circulation flow network with flow conservation constraints for each node and binary constraints on all edges, except the edge between the end and source node ($e^{I}_{E,S}$), whose flow is the total number of intervention groups selected. Beside these two types of constraints, an exclusivity constraint constrains the sum over all incoming edges of all intervention nodes related to the same object to be equal or smaller than one assuring that, not more than one intervention is selected per object. This is required in order to guarantee the validity of the benefit value associated with each intervention. If two interventions would be selected, the object would already be enhanced by the first intervention and the risk reduction of the second intervention would be less than attributed to the intervention in the model. If the combination of two interventions on an object is possible, this combination has to be implemented as a separate possible intervention on the object.

This formulation of the intervention layer ensures that the fixed intervention costs are only selected once for a group of interventions. It also enables risk-reduction to be related directly to the intervention on the object when the risk is assessed per object, i.e. only one object can fail at a time.

4.3.5 Relationship between the intervention layer and the traffic state layers

An intervention, which is selected in the intervention layer and that affects traffic, is a source of user costs. Since the user costs depend on the traffic state, under which interventions are executed, the duration of the selected interventions form the source flow in the traffic state layers, which models the duration of each traffic state that has to be applied within a time period. The interlayer edges ($e^{I\text{-}TS}_{v}$), therefore, are used to transform the selection of the interventions in the intervention layer into the length of time required to execute the intervention if a specific traffic state is used, i.e. intervention durations. This is done by the introduction of intervention duration constraints that form additional flow equilibriums at the intervention nodes in the intervention layers considering the inflow edges of these nodes and the outgoing interlayer edges. The duration is estimated by multiplying the sum of all inflows with the duration of the intervention. For example, assuming intervention 5 is executed in traffic state

3. The value of 5 on the interlayer edge of intervention 5 to traffic state 3 ($e^{I\text{-}TS3}{}_{5}$) is equal to the sum of the incoming edges in the intervention layer ($e^{I}{}_{S,5}$ and $e^{I}{}_{4,5}$), which have a value of 1 and 0, respectively, multiplied by the duration of the intervention that is 5. This can be seen in the equation below.

$$\left(e^{I}_{S,5}+e^{I}_{4,5}\right)*Duration_5=(1+0)*5\ time\ units=5\ time\ units=e^{I-TS3}_{5}$$

The costs of interlayer edges are 0 since they do not have any costs or benefits related to them, and are only used to ensure the mathematical consistency between the different layers.

4.3.6 Traffic state layers

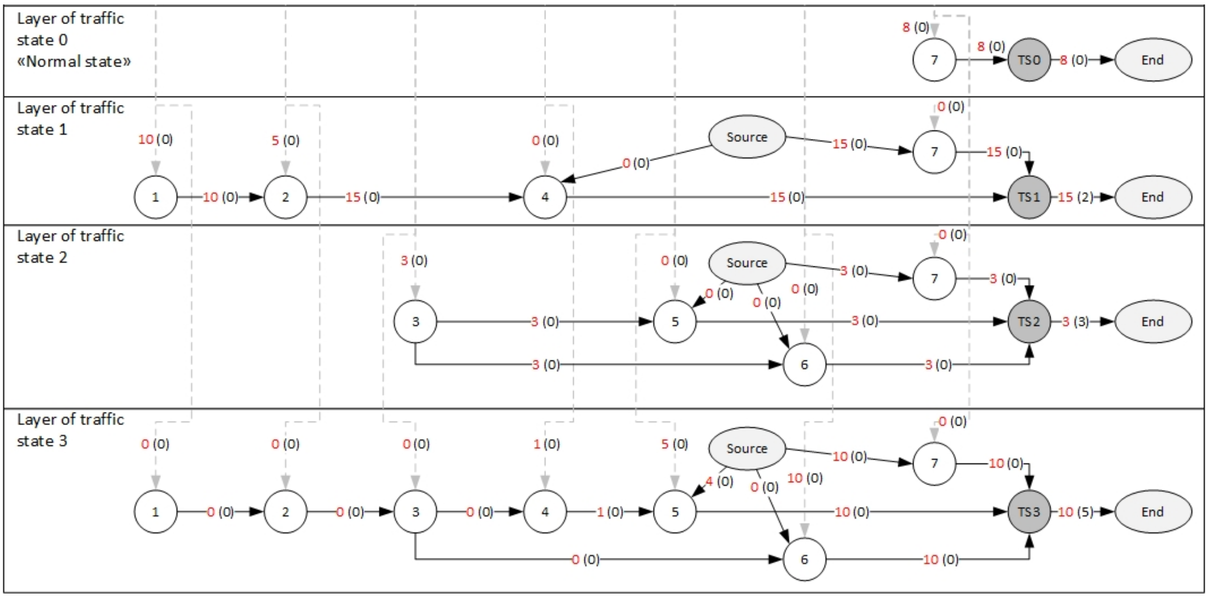

Figure 7. Illustration of the traffic state layers

Each traffic state is represented in the network model with its own layer. The description of a traffic state layer at this point is valid for all traffic state layers, since all traffic state layers have the same structure and characteristics. The network within a traffic state layer consists of all intervention nodes v whose interventions can be executed with the traffic state in focus, and the inlayer edges $e^{TS}_{u,v}$. For example in Figure 7, the intervention nodes *1*, *2*, *4*, and *7* can be executed when the network is in traffic state *1* and are, therefore, nodes in the traffic state 1 layer. The flow across the interlayer edges from the intervention layer to the traffic state layer, which indicates the amount of time the network is required to be in the specified traffic state to execute the interventions, are the flow sources of the traffic state layer. They then continues through the intervention nodes to the traffic state node. The flow at the traffic state node equals the total duration a specific traffic state has to be put in place in order to execute the interventions. In the example in Figure 7, traffic state 1 sums up the durations of intervention 1, 2, and 4 if they are selected and executed within traffic state 1.

Before the actual network structure in a traffic state layer is explained, the relationship between the individual intervention durations and the total traffic state duration is considered in more detail. Section 3.4.3 stated the topological dependencies between interventions from different intervention types. There, it was identified that interventions from type 1 have to be executed in series in time when they are within the same route, e.g. a track and a catenary intervention. Further, interventions of type 2 cannot be executed at the same time as interventions of type 1, e.g. a major bridge intervention and the overlaying track. Multiple interventions of type 2 and type 3, however, can be executed in parallel in time, thus reducing the total duration.

Figure 8 illustrates how interventions from these three types can be arranged to find the total required traffic state duration. Each block refers to an intervention, where blue refers to type 1 interventions, red to type 2 interventions, and green to type 3 interventions. The x-axis indicates the time while the y-axis indicates parallel execution. All interventions within one traffic state can be arranged according to the rules from Section 3.4.3. Type 1 and type 2 interventions are put in series order to each other while different type 2 interventions can be piled on each other, which means that they can be executed in parallel. Type 3 interventions can be piled on top of any type 1 or type 2 interventions. The maximal required duration for executing all these interventions is then indicated by the most right element in the graph, e.g. intervention H.

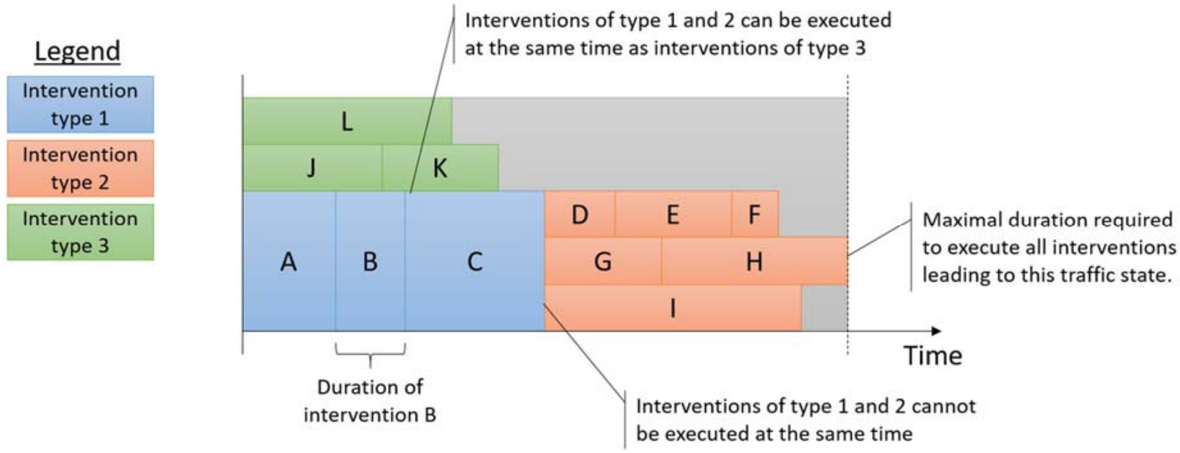

Figure 8. Relationship between the traffic state duration and the individual intervention durations considering different intervention types

This set-up of the interventions within a traffic state is used to construct the network structure of the traffic state layers. Each intervention is connected with its successors by an inlayer edge $e^{TS}_{u,v}$. The last interventions in a row are connected with the traffic state node. Figure 9 shows the resulting network for the example traffic state in Figure 9. The idea is that the flow on an outgoing edge of an intervention is equal to the flow on the incoming edges plus the duration of the intervention itself. Thus, all the individual durations are summed from left to right.

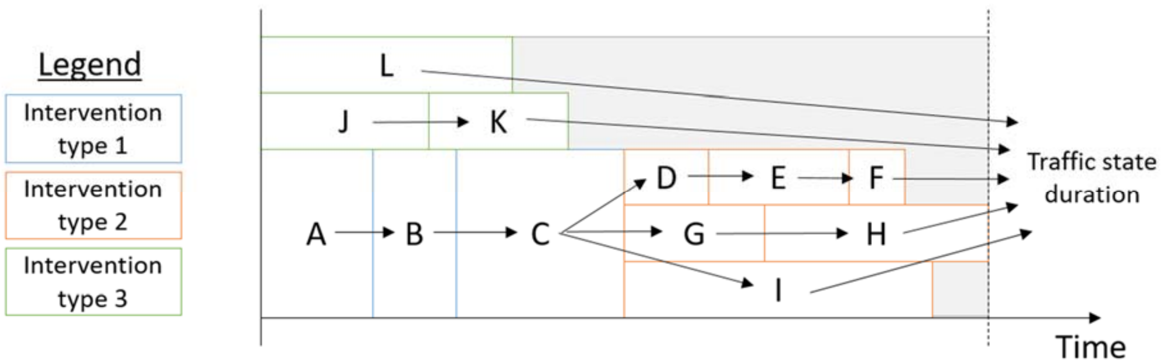

Figure 9. Network structure of a traffic state layer

Coming back to the network model presented in Figure 5, intervention *7* can be executed parallel to the other interventions in all traffic states while in traffic state 2 and 3 intervention *6* can be executed parallel to intervention *5,* or interventions *4* and *5*.

To be able to formulate feasible flow conservation constraints within the traffic state layers, the node equilibriums have to be formulated in a form similar to the interlayer edges and one additional source node has to be introduced. When a node represents a split in the network flow, each outgoing flow has to be equal to the total inflow including the inflow from the interlayer edges. This is achieved by first multiplying all inflows by the ratio of outgoing and incoming edges within the flow conversation constraint, and second formulate an equal flow constraint that ensures that all outflows are equal. For example, intervention *3* in traffic state 3 of Figure 7 is connected to both, intervention *4* and *6*. The inflow of inlayer edge $e^{TS3}_{2,3}$ and interlayer edge $e^{I\text{-}TS3}_{3}$ is, therefore, multiplied by two (e.g. 2 out / 1 in = 2). The equal flow constraint then assures that the outflow on edge $e^{TS3}_{3,4}$ is equal to the outflow on edge $e^{TS3}_{3,6}$. The same is used for the merging of flows at the traffic state node, e.g. *TS3*. All incoming flows have to be equal and are divided by 3 (e.g. 1 out / 3 in = ⅓).

The source node is connected to the last intervention of each branch. The non-negative flow on the edge $e^{TS}_{S,v}$ from the source node to the intervention node *v* fills up the flow on each branch so that the flows on all branches are equal. This is shown by the grey area in Figure 9. Since only the maximum duration is of interest, all non-maximum branches have to be filled in order to get equal flows. With this approach of summing the total duration of a traffic state, the last edge in each traffic state layer ($e^{TS}_{TS,E}$) has a flow equal to the duration required to execute all interventions within the traffic state in focus. This is, therefore, the only edge with a cost, which represents the costs of the particular traffic state per time unit.

The flow increases from node to node due to the additional duration required to execute the new intervention. The outflows from intervention *3* are both equal to the sum of all inflows into intervention node 3 and on the merge into node *TS3* each inflow equals the outflow. The source edge to intervention node *5* is 4 because the branch *3-4-5-TS3* would be only 24 instead of the 28 from branch *3-6-TS3*. The total user costs due to traffic state 3 would be 28 multiplied by the unit cost of traffic state 3.

Regarding the situation shown in Figure 7, traffic state layer 1 contains the execution of intervention 1 and 2. The flows on the interlayer edges to these two interventions are equal to the duration of the interventions, i.e. 10 and 5 time units. All other interlayer edges to the traffic state 1 layer are 0. The outflow from intervention 2 is the sum of the durations of both interventions, i.e. 15 time units. In order to have an equal flow on all incoming flows of the traffic state node *TS1*, the flow between intervention 7 and *TS1* has to be 15 time units as well, and therefore, intervention 7 requires 15 time units on the inflow from the source node. The traffic state 2 layer, where intervention 3 is executed, it can be seen that the inflow on intervention 3 of 3 time units is equal to both outflows to intervention 4 and 6. In the traffic state 3 layer, the flows from the source to intervention 5 and intervention 7 are 4 and 10 time units, respectively, in order to equal the maximum flow of 10 time units that is due to intervention 6.

It is noted that even though the interventions are ordered and positioned in respect of time, this does not state the actual order in which the interventions will be executed. It is only done in order to estimate the total required duration of the traffic state and the user costs related to this duration.

4.3.7 Side constraints

The network model formulated covers almost the entire problem. It does not include the constraints discussed in section 4.2. These constraints are formulated together with the exclusivity constraints mentioned in 4.3.4 as side constraints as described in section 4.2.

5 General mathematical model

5.1 Objective

A minimal cost flow model, on which this model is based on, is usually formulated with a cost minimisation objective. The objective function in this model, however, is stated as a net benefit maximisation as shown in equation 1. The transformation from a minimisation to a maximisation problem is feasible and only requires reversing the signs of the weights. Equation 7 shows the objective function of the network flow model.

$$Max\ Z = \sum_{v=1}^{V}\sum_{u=1}^{U} \delta_{u,v}^{I} * \left(rr_{u,v} - c_{u,v}\right) - \sum_{ts=1}^{TS} f_{TS,E}^{ts} * c_{ts} \qquad 7$$

Where $\delta_{u,v}^{I}$ is a binary variable that is 1 if the edge between the nodes *u* and *v* is part of the optimal path and 0 otherwise, $rr_{u,v}$ and $c_{u,v}$ are the benefit and cost of the edge between the nodes *u* and *v*.

An exception of the binary constraint for $\delta_{u,v}^{I}$ is $\delta_{E,S}^{I}$, which is the circle closing edge between the end and the start node, whose value is equal to the number of selected interventions.

5.2 Flow conservation constraint

The central constraint in a flow network is the flow conservation constraint. Equations 8 and 9 show the flow conservation constraints for the intervention selection and the traffic state layers.

$$\sum_{v=1}^{V} \delta_{u,v}^{I} - \sum_{v=1}^{V} \delta_{v,u}^{I} = 0\ \forall u \qquad 8$$

$$\sum_{v=1}^{V} f_{u,v}^{ts} - \sum_{v=1}^{V} \left(f_{v,u}^{ts} + f_{u}^{I-ts}\right) * a_{v,u}^{ts} = 0\ \forall u, ts \qquad 9$$

Where f_{v}^{I-ts} is the flow on the interlayer edge of node *u* and $a_{v,u}^{ts}$ is the ratio between the regular outgoing and regular incoming edges of node *u* in the traffic state layer *ts*. Regular refers to all inlayer edges except the edges starting at the source node.

5.3 Duration transformation constraint

The duration transformation constraints shown in equation 10 transforms the intervention selection from the intervention layer into the duration sources of the traffic state layers.

$$\sum_{u=1}^{U} \delta_{u,v}^{I} * d_{v} = \sum_{ts=1}^{TS} f_{v}^{I-ts}\ \forall u, n \qquad 10$$

Where d_v is the duration of the intervention of node *v*.

5.4 Exclusivity constraint

The exclusivity constraint in equation 11 ensures that only one intervention per object is selected.

$$\sum_{u=}^{U}\sum_{v=1}^{V_n} \delta_{u,v}^{I} \leq 1 \; \forall n \qquad 11$$

Where V_n is the set of all nodes v referring to an intervention on object *n*.

5.5 Equal flow constraint

A diverged flow in the traffic state layer has to be equalised on all branches, which is achieved with the constraint in equation 12.

$$f_{u,v}^{ts} = f_{u,w}^{ts} \; \forall u, v, w, ts \qquad 12$$

5.6 Organisational constraint

Organisational constraints, such as a budget constraint, are formulated according to equation 13.

$$\sum_{v=1}^{V}\sum_{u=1}^{U} \delta_{u,v}^{I} * c_{u,v} \leq \Omega_{max} \qquad 13$$

Where Ω_{max} is the budget limitation.

5.7 Structural constraint

Structural constraints, i.e. those due to structural dependencies, ensure that the mandatory intervention is selected when its initial intervention is selected.

$$\sum_{u=1}^{U} \delta_{u,v}^{I} - \sum_{u=1}^{U} \delta_{u,w}^{I} \leq 0 \; \forall (v, w) \qquad 14$$

Where the pair *(v,w)* refers to a pair of intervention, where *w* is a mandatory intervention of *v*.

6 Example

In the example, intervention programs are developed for a small part of the Irish railway infrastructure network located in Dublin Ireland. Only a small part was chosen in order to construct the model in a clear and understandable way, to hold the computational size small, and to be able to evaluate different approaches in terms of finding the optimal intervention program. The intervention programs are developed for one year and with and without the consideration of a budget limitation. The intervention programs are developed using three different approaches, 1) exhaustive search, 2) decision rules, and 3) a network flow model. The exhaustive search approach explicitly tests all possible combination of interventions. The decision rules approach use a set of rules to help focus the search resulting in only a sub set of the possible intervention programs to be tested. The network flow model approach is used to construct a network representation of the entire system in a way that an integer linear program can be formulated to find the optimal intervention program, using the simplex method and branch and bound search algorithms. Section 6.1 provides all base information for the example. This includes the railway infrastructure network, the possible interventions, the traffic on the network, and the risk related to the objects. Section 6.2 contains a summary of the different approaches used. Section 6.3 contains the results from the different models. Section 6.4 contains the discussion of the results.

6.1 Example situation

In the example, the intervention program is developed for the railway infrastructure between *Connolly Station* and *Grand Canal Dock* in Dublin, Ireland. Figure 10 provides an overview of the area that includes the additional stations *Tara Street* and *Dublin Pearse*. It is a roughly 2'200 meter long double track line in the centre of Dublin. The track layout is shown in Figure 11, which can be seen in a higher resolution in Annex A. Even though, it is small compared with the railway network of the whole country, it is sufficient to show almost all effects occurring in a railway infrastructure network.

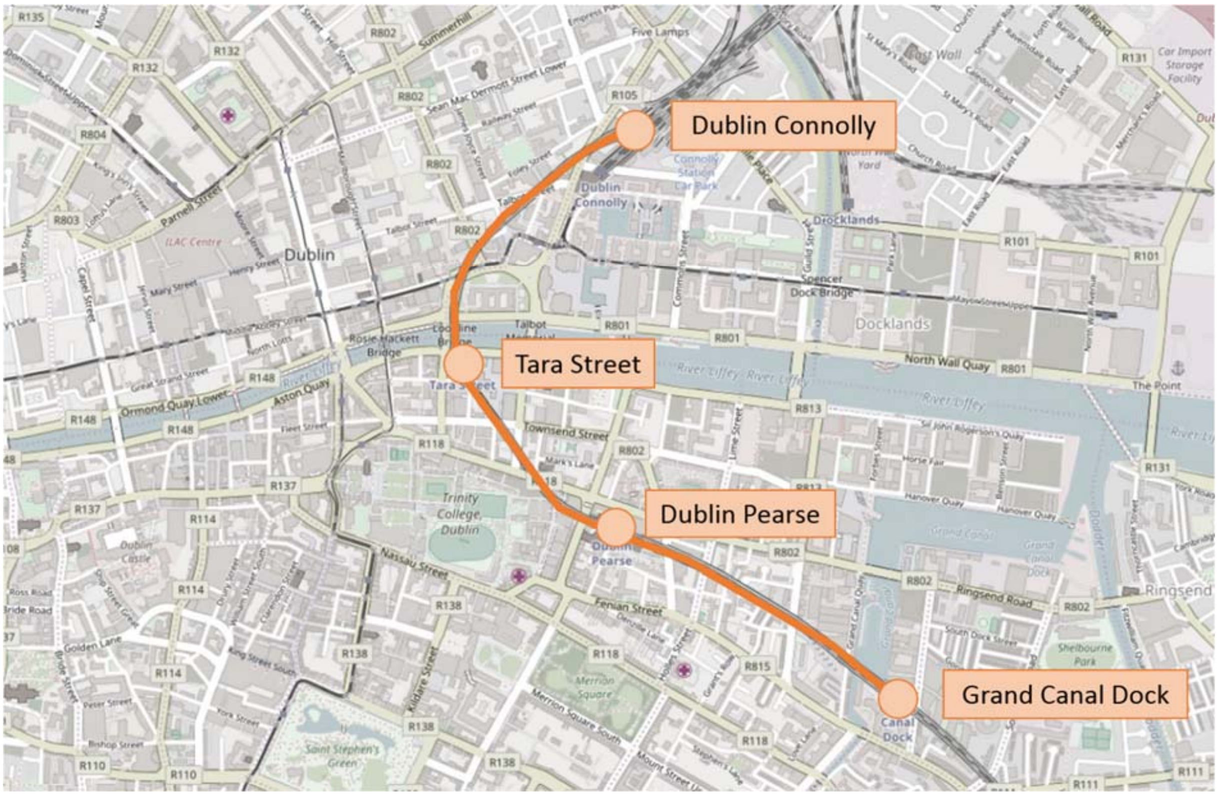

Figure 10. Area of the example (map source: OpenStreetMap Contributors, 2017)

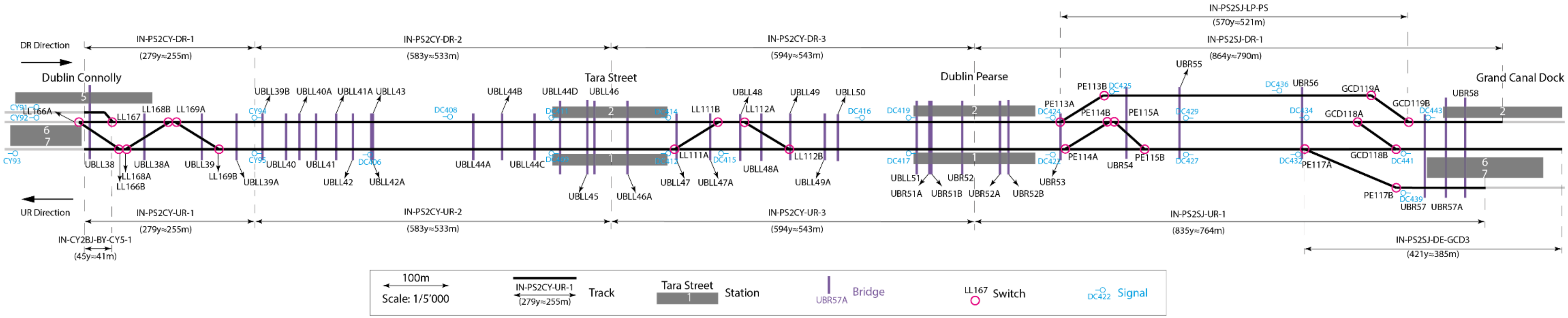

Figure 11. Network layout of the example area

In order to be able to develop intervention programs considering objects of different categories, while not increasing the example by too many objects, only the three major object categories of tracks, switches, and bridges are considered. The further inclusion of objects of other categories, e.g. safety and signalling infrastructure objects, would have led to an over-proportional increase in computational complexity during this early exploratory phase.

The following subsections provide all the base information needed for developing optimal intervention programs. All objects of the example area are identified and characterised in subsection 6.1.1. The risk related to every object and state is estimated build on the list of objects (6.1.2). Subsection 6.1.3 introduces all possible interventions considered in the example with their characteristics, while 6.1.4 works out the different dependencies in the example area accordingly to the dependencies introduced in section 3.4. The last two subsections provide the information about the traffic windows considered (6.1.5) and the user costs related to them (6.1.6).

6.1.1 Objects

The example considers the object categories track, switches, and bridges along the area of the example. They are summarised in Table 6.

Table 6. Objects included in the example

Category	Number of objects	Extend
Track	11	5’163 m
Switch	23	-
Bridge	39	16’763 m^2

The track is the first object category. It has to be divided due to its nature of being a continuous object type and the requirement of discrete objects within this model. Figure 12 shows the track segmentation and the current state of the objects. The figure is shown in a higher resolution in Annex A. In general, the tracks are divided at the station, since stations are important operation locations. A further division in between of two stations makes sense when a switch section exists close to one station, which is guarded by signals outside of the station. For example, the track between *Dublin Connolly* and *Tara Street* is divided into two segments, one containing the switches in the run-up of *Dublin Connolly* and one from *Tara Street* up to the signals guarding this switch section. Additional tracks are bound at the switches in the main line where the additional tracks start or end.

In the example, the double track line between *Connolly Station* and *Pearse* is divided into six track segments, three for the up line and three for the down line. Between *Pearse* and *Grand Canal Dock*, there exists an additional loop track beside the standard double track line. The down line track, the up lane track and the loop track are one track segment each. The dead-end track in the *Grand Canal Dock* station and the platform 5 track in *Dublin Connolly* are further track segments and the 10th and 11th segments within this area. The track segments are listed in Table 7 with their related length and current state.

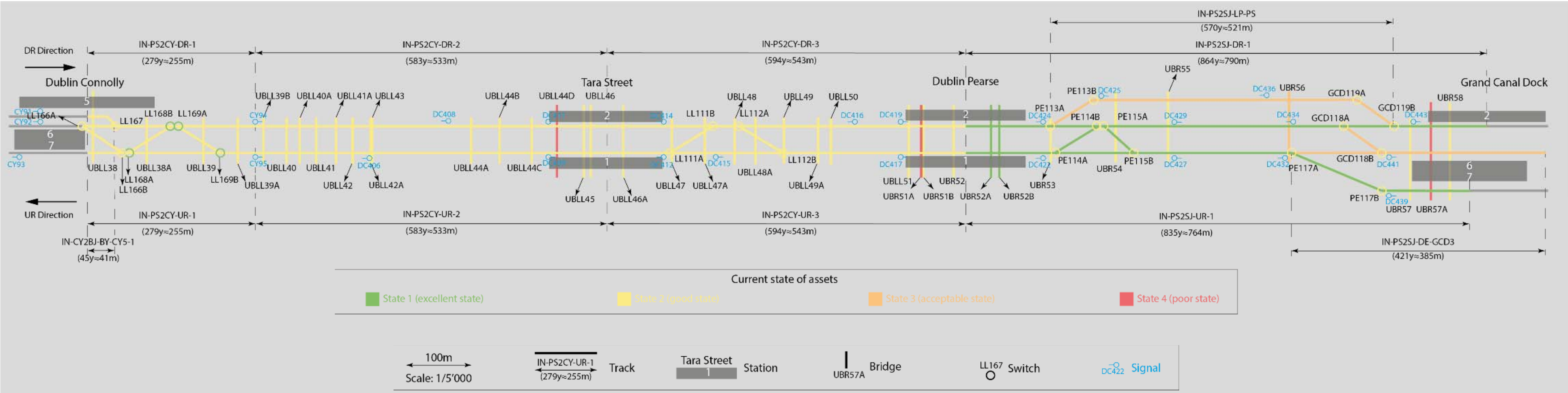

Figure 12. Objects in the example including their current state

Table 7. Track segments

ID	Serial Number	Length [m]	Current state
T_1	IN-PS2CY-DR-1	255	2 – Good
T_2	IN-PS2CY-UR-1	255	2 – Good
T_3	IN-PS2CY-DR-2	533	2 – Good
T_4	IN-PS2CY-UR-2	533	2 – Good
T_5	IN-PS2CY-DR-3	543	2 – Good
T_6	IN-PS2CY-UR-3	543	2 – Good
T_7	IN-PS2SJ-DR-1	790	1 – Excellent
T_8	IN-PS2SJ-UR-1	764	1 – Excellent
T_9	IN-PS2SJ-LP-PS	521	3 – Acceptable
T_{10}	IN-PS2SJ-DE-GCD	385	3 – Acceptable
T_{11}	IN-CY2BJ-BY-CY5-1	41	2 – Good

Figure 12 includes further the switches and bridges considered in the example. A total of 23 switches and 39 bridges lay within the area of the example. Table 8 and Table 9 list the switch and bridge objects with their required information. A common information for both object categories refers to the adjusted track segment, which identifies the location of the object in respect to the different track segments. Since bridges span both routes of a double track line, they are related to a pair of track segments. Bridges are further divided into their construction material, e.g. concrete, steel, and masonry.

Table 8. Switches

ID	Serial Number	Adjusted track segment	Current state
S_1	LL166A	T_1	2 – Good
S_2	LL166B	T_2	2 – Good
S_3	LL167	T_1	2 – Good
S_4	LL168A	T_2	1 – Excellent
S_5	LL168B	T_1	1 – Excellent
S_6	LL169A	T_1	1 – Excellent
S_7	LL169B	T_2	1 – Excellent
S_8	LL111A	T_6	2 – Good
S_9	LL111B	T_5	2 – Good
S_{10}	LL112A	T_5	2 – Good
S_{11}	LL112B	T_6	2 – Good
S_{12}	PE113A	T_7	2 – Good
S_{13}	PE113B	T_9	2 – Good
S_{14}	PE114A	T_8	2 – Good
S_{15}	PE114B	T_7	2 – Good
S_{16}	PE115A	T_7	2 – Good
S_{17}	PE115B	T_8	2 – Good
S_{18}	PE117A	T_8	2 – Good
S_{19}	PE117B	T_8	2 – Good
S_{20}	GCD118A	T_7	2 – Good
S_{21}	GCD118B	T_{10}	2 – Good
S_{22}	GCD119A	T_9	2 – Good
S_{23}	GCD119B	T_7	2 – Good

Table 9. Bridges

ID	Serial Number	Adjusted track segment	Surface area [m^2]	Construction type	Current state
B_1	UBLL38	T_1, T_2, T_{11}	720	Masonry	2 – Good
B_2	UBLL38A	T_1, T_2	1130	Concrete	2 – Good
B_3	UBLL39	T_1, T_2	470	Masonry	2 – Good
B_4	UBLL39A	T_1, T_2	320	Steel	2 – Good
B_5	UBLL39B	T_3, T_4	372.4	Steel	2 – Good
B_6	UBLL40	T_3, T_4	166.6	Steel	2 – Good
B_7	UBLL40A	T_3, T_4	166.6	Masonry	2 – Good
B_8	UBLL41	T_3, T_4	350	Steel	2 – Good
B_9	UBLL41A	T_3, T_4	500	Steel	2 – Good
B_{10}	UBLL42	T_3, T_4	250	Masonry	2 – Good
B_{11}	UBLL42A	T_3, T_4	350	Steel	2 – Good
B_{12}	UBLL43	T_3, T_4	1410	Masonry	2 – Good
B_{13}	UBLL44A	T_3, T_4	500	Steel	2 – Good
B_{14}	UBLL44B	T_3, T_4	450	Masonry	2 – Good
B_{15}	UBLL44C	T_3, T_4	400	Steel	2 – Good
B_{16}	UBLL44D	T_3, T_4	640	Steel	4 – Poor
B_{17}	UBLL45	T_3, T_4	230	Steel	2 – Good
B_{18}	UBLL46	T_3, T_4	230	Masonry	2 – Good
B_{19}	UBLL46A	T_5, T_6	960	Steel	2 – Good
B_{20}	UBLL47	T_5, T_6	320	Steel	2 – Good
B_{21}	UBLL47A	T_5, T_6	600	Masonry	2 – Good
B_{22}	UBLL48	T_5, T_6	330	Masonry	2 – Good
B_{23}	UBLL48A	T_5, T_6	460	Masonry	2 – Good
B_{24}	UBLL49	T_5, T_6	450	Steel	2 – Good
B_{25}	UBLL49A	T_5, T_6	650	Steel	2 – Good
B_{26}	UBLL50	T_5, T_6	720	Steel	2 – Good
B_{27}	UBLL51	T_5, T_6	270	Steel	2 – Good
B_{28}	UBR51A	T_5, T_6	765	Steel	4 – Poor
B_{29}	UBR51B	T_5, T_6	192	Steel	2 – Good
B_{30}	UBR52	T_5, T_6	110	Steel	2 – Good
B_{31}	UBR52A	T_7, T_8	160	Masonry	1 – Excellent
B_{32}	UBR52B	T_7, T_8	240	Concrete	1 – Excellent
B_{33}	UBR53	T_7, T_8	345	Masonry	2 – Good
B_{34}	UBR54	T_7, T_8, T_9	345	Masonry	2 – Good
B_{35}	UBR55	T_7, T_8, T_9	136	Masonry	2 – Good
B_{36}	UBR56	T_7, T_8, T_9	425	Masonry	3 – Acceptable
B_{37}	UBR57	T_7, T_8, T_{10}	187	Masonry	2 – Good
B_{38}	UBR57A	T_7, T_8, T_{10}	187	Concrete	4 – Poor
B_{39}	UBR58	T_7, T_8, T_{10}	255	Masonry	2 – Good

6.1.2 Risk

In order to be able to estimate the change in risk due to deterioration and interventions executed on the objects, the risk has to be assessed. The risk assessment process developed in the deliverable 3.2 was developed for different levels (e.g. object, section, route, and line) in respect with the consideration of multiple objects to fail at the same time (Papathanasiou et al., 2016). In this example, the risk is assessed on the object level, which requires the risk estimation for each object and state. Table 10, Table 11, and Table 12 show the risk estimated for every object and each state. The risk estimation is part of the deliverable 3.6 (Papathanasiou et al., 2018), to which is referenced here for the more detailed estimations.

Table 10. Risk related to track segments [Euros]

ID	Serial Number	State 1 (excellent)	State 2 (good)	State 3 (acceptable)	State 4 (poor)
T_1	IN-PS2CY-DR-1	6'277	62'769	188'306	1'077'730
T_2	IN-PS2CY-UR-1	6'277	62'769	188'306	1'077'730
T_3	IN-PS2CY-DR-2	6'535	65'348	196'043	1'321'163
T_4	IN-PS2CY-UR-2	6'535	65'348	196'043	1'321'163
T_5	IN-PS2CY-DR-3	6'489	64'891	194'673	1'272'209
T_6	IN-PS2CY-UR-3	6'489	64'891	194'673	1'272'209
T_7	IN-PS2SJ-DR-1	6'808	68'085	204'254	1'459'020
T_8	IN-PS2SJ-UR-1	6'767	67'669	203'008	1'437'856
T_9	IN-PS2SJ-LP-PS	6'230	62'305	186'914	1'220'734
T_{10}	IN-PS2SJ-DE-GCD	1'142	11'415	34'246	114'154
T_{11}	IN-CY2BJ-BY-CY5-1	506	5'057	15'170	50'565

Table 11. Risk related to switches [Euros]

ID	Serial Number	State 1 (excellent)	State 2 (good)	State 3 (acceptable)	State 4 (poor)
S_1	LL166A	8'542	101'014	293'571	1'432'348
S_2	LL166B	8'542	101'014	293'571	1'432'348
S_3	LL167	8'542	101'014	293'571	1'432'348
S_4	LL168A	8'542	101'014	293'571	1'432'348
S_5	LL168B	8'542	101'014	293'571	1'432'348
S_6	LL169A	8'542	101'014	293'571	1'432'348
S_7	LL169B	8'542	101'014	293'571	1'432'348
S_8	LL111A	7'092	78'982	218'675	1'153'207
S_9	LL111B	7'092	78'982	218'675	1'153'207
S_{10}	LL112A	7'092	78'982	218'675	1'153'207
S_{11}	LL112B	7'092	78'982	218'675	1'153'207
S_{12}	PE113A	6'808	75'470	208'123	1'129'964
S_{13}	PE113B	6'138	66'297	178'683	1'041'453
S_{14}	PE114A	6'804	74'744	204'511	1'102'878
S_{15}	PE114B	6'804	74'744	204'511	1'102'878
S_{16}	PE115A	6'808	75'470	208'123	1'129'964
S_{17}	PE115B	6'808	75'470	208'123	1'129'964
S_{18}	PE117A	6'804	74'744	204'511	1'102'878
S_{19}	PE117B	6'808	75'470	208'123	1'129'964
S_{20}	GCD118A	6'808	75'470	208'123	1'129'964
S_{21}	GCD118B	6'535	71'162	193'198	1'070'946
S_{22}	GCD119A	6'138	66'297	178'683	1'041'453
S_{23}	GCD119B	6'806	75'107	206'317	1'116'421

Table 12. Risk related to bridges [Euros]

ID	Serial Number	State 1 (excellent)	State 2 (good)	State 3 (acceptable)	State 4 (poor)
B_1	UBLL38	0	27'340	521'126	51'272'150
B_2	UBLL38A	0	0	34'932	18'527'602
B_3	UBLL39	0	19'428	365'771	33'636'327
B_4	UBLL39A	0	14'865	277'646	23'112'798
B_5	UBLL39B	0	0	15'914	6'110'853
B_6	UBLL40	0	11'077	202'233	12'403'148
B_7	UBLL40A	0	0	9'363	2'736'668
B_8	UBLL41	0	17'622	326'172	25'327'612
B_9	UBLL41A	0	22'975	427'540	35'898'330
B_{10}	UBLL42	0	14'053	258'594	18'280'467
B_{11}	UBLL42A	0	0	15'201	5'743'595
B_{12}	UBLL43	0	55'449	1'042'506	100'027'352
B_{13}	UBLL44A	0	22'975	427'540	35'898'330
B_{14}	UBLL44B	0	21'190	393'751	32'374'757
B_{15}	UBLL44C	0	19'406	359'962	28'851'185
B_{16}	UBLL44D	0	0	24'802	10'499'196
B_{17}	UBLL45	0	13'605	252'644	16'959'412
B_{18}	UBLL46	0	13'605	252'644	16'959'412
B_{19}	UBLL46A	0	0	34'988	15'745'743
B_{20}	UBLL47	0	16'509	305'105	23'145'518
B_{21}	UBLL47A	0	0	23'088	9'841'172
B_{22}	UBLL48	0	16'865	311'838	23'848'120
B_{23}	UBLL48A	0	0	18'648	7'546'107
B_{24}	UBLL49	0	21'132	392'637	32'279'349
B_{25}	UBLL49A	0	0	24'673	10'660'838
B_{26}	UBLL50	0	30'732	574'434	51'249'613
B_{27}	UBLL51	0	14'997	279'005	19'720'881
B_{28}	UBR51A	0	32'598	612'299	54'499'698
B_{29}	UBR51B	0	7'448	25'607	3'561'347
B_{30}	UBR52	0	0	7'919	1'809'364
B_{31}	UBR52A	0	6'574	22'608	2'973'274
B_{32}	UBR52B	0	8'759	30'104	4'443'456
B_{33}	UBR53	0	0	11'763	5'602'012
B_{34}	UBR54	0	0	11'806	5'602'088
B_{35}	UBR55	0	0	7'126	2'211'361
B_{36}	UBR56	0	0	13'597	6'899'974
B_{37}	UBR57	0	0	8'268	3'038'763
B_{38}	UBR57A	0	0	8'639	3'039'681
B_{39}	UBR58	0	0	10'161	4'142'885

6.1.3 Interventions

The interventions considered in the example are shown in Table 13, Table 14, and Table 15 for tracks, switches, and bridges, respectively. The type indicated refers to the different intervention types identified in Table 2 in chapter 3.3. All track interventions are continuous interventions (type 1), while the bridge and switch interventions belong to mayor on-track interventions (type 2). This classification means that all interventions require track occupancy and lead to the unavailability of the track for train traffic while executing the interventions. Each intervention is applied on a certain state of the object, restores the state to a certain state, has costs associated per unit, and needs a particular time to execute, which is expressed through either an efficiency value (units per h) or the duration per asset (hours per asset).

Table 13. Interventions for tracks

ID	Intervention	Type	Applied on state	Restored state	Cost	Duration
I_{T1}	Tamping	1	2	1	€ 7.5 / m	457 m / h
I_{T2}	Ballast Cleaning	1	3	1	€ 1.9 / m	119 m / h
I_{T3}	Track Renewal	1	4	1	€ 745.6 / m	119 m / h

Table 14. Interventions for switches

ID	Intervention	Type	Applied on state	Restored state	Cost	Duration
I_{S1}	Manual Grinding	2	2	1	€ 10,000 / asset	3 h / asset
I_{S2}	Welding	2	3	1	€ 10,000 / asset	3 h / asset
I_{S3}	Renewal	2	4	1	€ 250,000 / asset	36 h / asset

Table 15. Interventions for bridge

ID	Intervention	Type	Applied on state	Restored state	Cost*	Duration
I_{B1}	Recoating	2	2	1	€ 250 / m^2	3.75 m^2 / h
I_{B2}	Strengthening	2	3	1	M: € 1000 / m^2	0.5 m^2 / h
					C: € 1000 / m^2	0.7 m^2 / h
					S: € 3000 / m^2	0.5 m^2 / h
I_{B3}	Major Repair / Renewal	2	4	1	M: € 8000 / m^2	72 h / asset
					C: € 7500 / m^2	
					S: € 5000 / m^2	

*note: M stands for masonry, C for concrete, and S for steel bridge.

The intervention costs provided by Irish Rail are not divided into fixed and variable costs. Assumptions are made for each object category whether the intervention costs are reduced by a grouped execution and by how much. Tracks and switches are more uniform object types meaning that they do not differ significantly from one object to the other. It is assumed that for track interventions 20% and for switch interventions 40% of the costs shown above relate to fixed costs, can be saved by grouping the same interventions on different categories. Bridges are bigger and more individual objects compared with the two other categories. Thus, it is assumed that there is no reduction in the intervention costs possible by grouping.

6.1.4 Dependencies

The three different dependencies identified in section 3.4 are structural, economical, and topological dependencies. Structural dependencies refers to interventions on objects requiring other interventions on other objects before traffic can run over the particular track segment again. With respect to the objects and interventions considered in this example, structural dependencies only exists between bridge renewal (I_{B3}) and track renewal (I_{T3}), where a bridge renewal requires the renewal of the adjusted track objects. Table 16 lists the structural dependencies.

Table 16. Structural dependencies

Initial intervention		Mandatory intervention	
Renewal of bridge:	1 (B_1-I_{B3})	Renewal of tracks:	1 (T_1-I_{T3}) 2 (T_2-I_{T3}) 11 (T_{11}-I_{T3})
Renewal of bridges:	2 (B_2-I_{B3}) 3 (B_3-I_{B3}) 4 (B_4-I_{B3})	Renewal of tracks:	1 (T_1-I_{T3}) 2 (T_2-I_{T3})
Renewal of bridges:	5 (B_5-I_{B3}) 6 (B_6-I_{B3}) 7 (B_7-I_{B3}) 8 (B_8-I_{B3}) 9 (B_9-I_{B3}) 10 (B_{10}-I_{B3}) 11 (B_{11}-I_{B3}) 12 (B_{12}-I_{B3}) 13 (B_{13}-I_{B3}) 14 (B_{14}-I_{B3}) 15 (B_{15}-I_{B3}) 16 (B_{16}-I_{B3}) 17 (B_{17}-I_{B3}) 18 (B_{18}-I_{B3})	Renewal of tracks:	3 (T_3-I_{T3}) 4 (T_4-I_{T3})
Renewal of bridges:	19 (B_{19}-I_{B3}) 20 (B_{20}-I_{B3}) 21 (B_{21}-I_{B3}) 22 (B_{22}-I_{B3}) 23 (B_{23}-I_{B3}) 24 (B_{24}-I_{B3}) 25 (B_{25}-I_{B3}) 26 (B_{26}-I_{B3}) 27 (B_{27}-I_{B3}) 28 (B_{28}-I_{B3}) 29 (B_{29}-I_{B3}) 30 (B_{30}-I_{B3})	Renewal of tracks:	5 (T_5-I_{T3}) 6 (T_6-I_{T3})
Renewal of bridges:	31 (B_{31}-I_{B3}) 32 (B_{32}-I_{B3}) 33 (B_{33}-I_{B3})	Renewal of tracks:	7 (T_7-I_{T3}) 8 (T_8-I_{T3})
Renewal of bridges:	34 (B_{34}-I_{B3}) 35 (B_{35}-I_{B3}) 36 (B_{36}-I_{B3})	Renewal of tracks:	7 (T_7-I_{T3}) 8 (T_8-I_{T3}) 9 (T_9-I_{T3})
Renewal of bridges:	37 (B_{37}-I_{B3}) 38 (B_{38}-I_{B3}) 39 (B_{39}-I_{B3})	Renewal of tracks:	7 (T_7-I_{T3}) 8 (T_8-I_{T3}) 10 (T_{10}-I_{T3})

The economical dependencies refer to economic benefit for the infrastructure owner due to grouping interventions on neighbouring objects, which reduces shared fixed costs. In the intervention section above, it is assumed that 20% of the total costs for track interventions and 40% of the total costs for switch interventions are fixed costs. This 20% and 40%, respectively, can be reduced when an intervention on one object is grouped with the same intervention on a physically close object. Table 17 lists the objects that have economical dependencies when interventions of the same type are executed on them. Economical dependencies are only possible for track interventions when the tracks are connected with each other topological. They are, therefore, listed in an ordered way in the table. Switch groups, on the other hand, have reductions in intervention costs that vary as a function of the number of switches in an area.

Table 17. Economical dependencies

Object group	Objects	Proportion of fix costs
Tracks, Up rail	$T_1 - T_3 - T_5 - T_7$ (continuos)	20 %
Tracks, Down rail	$T_2 - T_4 - T_6 - T_8$ (continuos)	20 %
Switches, Connolly	S_1, S_2, S_3, S_4, S_5, S_6, S_7	40 %
Switches, Tara street	S_8, S_9, S_{10}, S_{11}	40 %
Switches, Pearse	S_{12}, S_{13}, S_{14}, S_{15}, S_{16}, S_{17}	40 %
Switches, Gran canal dock	S_{18}, S_{19}, S_{20}, S_{21}, S_{22}, S_{23}	40 %

Topological dependencies refer to the connectivity of objects with respect to the loss in service when interventions are executed on them. If interventions on the objects lead to the closure of the same track segment and it is possible to execute the interventions at the same time, then the loss in service is reduced due to the shorter time required to execute the interventions together when compared with the execution of the interventions, individually. Table 18 summarises the topological dependencies by first stating the topological relation between objects in reference of the different track segments, and second by differentiate all possible interventions within a track segment according to their type. In section 3.4, it was elaborated that type 1 interventions cannot be executed simultaneously with other interventions within the same track segment. Multiple type 2 interventions, however, can be executed at the same time because they are stationary at one location of the entire segment and it is assumed that they do not overlap in space.

Table 18. Topological dependencies

Track segment	Objects in this segment	Interventions of type 1	Interventions of type 2
T_1	T_1, S_2, S_4, S_7, B_1	All on T_1	All interventions on S_2, S_4, S_7, B_1
T_2	T_2, S_1, S_3, S_5, B_1	All on T_2	All interventions on S_1, S_3, S_5, B_1
T_3	T_3	All on T_3	All interventions on S_1, S_3, B_2, B_4
T_4	T_4, B_2	All on T_4	All interventions on B_2
T_5	T_5, B_2	All on T_5	All interventions on B_2
T_6	T_6, S_8, S_{11}	All on T_6	All interventions on S_8, S_{11}
T_7	T_7, S_9, S_{10}	All on T_7	All interventions on S_9, S_{10}
T_8	T_8, S_{14}, S_{17}, S_{18}, S_{19}, B_3, B_4	All on T_8	All interventions on S_{14}, S_{17}, S_{18}, S_{19}, B_3, B_4
T_9	T_9, S_{12}, S_{15}, S_{16}, S_{20}, S_{23}, B_3, B_4	All on T_9	All interventions on S_{12}, S_{15}, S_{16}, S_{20}, S_{23}, B_3, B_4
T_{10}	T_{10}, S_{13}, S_{22}	All on T_{10}	All interventions on S_{13}, S_{22}
T_{11}	T_{11}, S_{21}	All on T_{11}	All interventions on S_{21}

6.1.5 Traffic windows

Interventions are executed during a certain time window that defines how much traffic is affected from the intervention execution. The traffic windows considered are shown in Table 19. The *weekend* and the *night* window are limited in their durations, while the *weekday* window is unlimited. Even though there are only five weekdays between two consecutive weekends and each day is followed by a night, weekdays are considered to be unlimited in their duration in order to ensure that each intervention can be executed within at least one possible time window. An intervention of more than one week will therefore, overestimate the loss in service, which however, is neglected here since the exact scheduling is not the aim of this work. It has to be considered that the actual working time per time window is less than the maximal time window length, as it is mentioned in section 3.5.

It is assumed that one hour is needed for set-up at the beginning and half an hour for clearing at the end of each time window independently of the intervention executed. Even though, this is a simplified assumption, the model would only be extended in size but not in complexity by the introduction of intervention specific values. Table 19 lists all traffic windows with their operational time, the maximal time window length, and the shorter working time per time window.

Table 19. Time windows considered

ID	Time window	Start and end time	Maximal time window length	Maximal work time
TW_1	weekday	-	Unlimited	Unlimited
TW_2	weekend	Saturday 00:00 – Monday 5:30	53.5 h	52 h
TW_3	Night	00:00 – 5:30	5.5 h	4 h

6.1.6 Traffic states and their unit costs

To be able to estimate the user costs, the delays due to loss in service and the value of travel time have to be determined. The latter is set to be 30.88 euros per hour (DESTination RAIL, 2018).

The delays are estimated based on expert opinions about the incurred delays per train due to changed traffic states together with the traffic in the example area as it is presented in National Transport Authority (2016) and in Aksentijevic et al. (2017). The delays are estimated for each combination of closed track segments and traffic windows. The different segment closures are referred to as traffic states of the network. Only the segment combinations are considered that either are necessary due to the objects spread (e.g. bridges spanning more than one track), or may improve the intervention program (e.g. due to economical or topological dependencies) compared with the intervention program where each object is considered individually. Within this example, the closure of each track segment separately and the combination of parallel segments leading to a full line closure between two stations are considered. An example for the second case is the simultaneously closure of track segments T_3 and T_6 that leads to a full closure of the line between *Tara Street* and *Dublin Pearse*. Table 20 provides the estimated delays in minutes per hour and the converted costs in euros per hour.

Table 20. Losses in level of service and user costs due to closed track segments

Closed track segment	TW_1 (weekday)		TW_2 (weekend)		TW_3 (night)	
	Delay [min/h]	Cost [Euro/h]	Delay [min/h]	Cost [Euro/h]	Delay [min/h]	Cost [Euro/h]
T_1	36’179	18'620	4’735	2'437	0	0
T_2	36’179	18'620	4’735	2'437	0	0
T_3	9’730	5'008	1’714	882	0	0
T_4	9’730	5'008	1’714	882	0	0
T_5	13’443	6'919	4’273	2'199	0	0
T_6	13’443	6'919	4’273	2'199	0	0
T_7	9’555	4'918	2’759	1'420	0	0
T_8	9’555	4'918	2’759	1'420	0	0
T_9	0	0	0	0	0	0
T_{10}	5’377	2'767	1’709	880	0	0
T_{11}	36’179	18'620	4’735	2'437	0	0
$T_1 + T_2 + T_3 + T_4$	80’966	41'671	25’716	13'235	0	0
$T_5 + T_6$	80’656	41'511	25’639	13'196	0	0
$T_7 + T_8 + T_9$	57’330	29'506	16’554	8'520	0	0

The traffic delay estimation considers the number of trains and passengers per day divided into peak and off-peak hours, as well as the occurring delays in case of a segment closure. The number of trains is derived from the current timetable. The number of passengers traveling in the trains is not know directly. It is estimated based on the passenger entries and exits in the station of *Dublin Pearse*, which are assumed to make 75 % of all passengers in trains passing the example network. Since there is no information available about the number of through passengers, the total number has to be assumed. Therefore, it is assumed that 75 % of all passengers in a train approaching or departing *Dublin Pearse* are going to leave or have entered the train, respectively in *Dublin Pearse*, and only 25 % is through traffic. This rough assumption can be justified with the important location of *Dublin Pearse* regarding the southbound traffic. Table 21 summarises the estimation of passengers per train.

Table 21. Estimation of passengers per train

	Total	Peak	Off-peak
Trains per year	87’869	33’500	54’369
Passengers per year: Enter and exit *Dublin Pearse* (75%) Through traffic (25%) Total (100%)	 10’764’402 3’588’134 14’352’536	 6’996’861 2’332’287 9’329’148	 3’767’541 1’255’847 5’023’388
Average passengers per train:		278.5	92.4

The occurring delays in case of a segment closure is based on expert opinion estimating the remaining capacity in the number of trains and delay per train running when each segment is closed separately. When the actual number of trains surpasses the decreased capacity, trains have to be cancelled between two stations, as they are in case of a full line closure. It is assumed that passengers from a cancelled train suffer 30 minutes of delay.

In order to clarify the delay values in Table 20, the following example shows the estimation of the delay for a *weekday* closure of track segment T_1. The expert opinion on a closure of this segment provide a capacity limit of 12 trains per hours (both directions) on the remaining open network, a delay of 30 minutes per cancelled train, and a delay of 5 minutes per remaining train. This capacity limit allows only 72 trains out of the 127 peak hour trains per day to run and leads to the cancellation of 55 trains. 11 out of the 143 off-peak hour trains per day have to be cancelled due to the capacity restriction leading to 132 remaining trains during the off-peak hours. In Table 22, the delays are calculated for the remaining peak and off-peak trains as well as for the cancelled peak and off-peak trains each using the formula shown in equation 15. Summed up, the total delay for a *weekday* closure of track segment T_1 equals to 36’179 minutes of delay per hours.

$$\frac{\frac{Number\ of\ trains}{Day} * \frac{Number\ of\ passengers}{Train} * \frac{Minutes\ of\ delay}{Passenger}}{18\ hour\ of\ operation} = \frac{Minutes\ of\ delay}{Hour} \qquad 15$$

Table 22. Delay calculation for closure of segment T_1 during the week.

	Total	Peak	Off-peak
Delay of passengers in remaining trains:			
Remaining trains		72	132
Passengers per train [passenger/train]		278.5	92.4
Delay per train [min/train]		5	5
Subtotal [min/hour]	8’957	5’570	3’387
Delay of passengers in cancelled trains:			
Cancelled trains		55	11
Passengers per train [passenger/train]		278.5	92.4
Delay per train [min/train]		30	30
Subtotal [min/day]	27’222	25’528	1’694
Subtotal			
Hours of operation [h/day]			
Total delay [Minutes per hour of closure]	36’179		

6.1.7 Summary

This section summarises the information from the former sections. The example area includes 73 objects (11 tracks, 23 switches, and 39 bridges) distributed along the 11 track segments used for estimating user costs. For each object category, three interventions are possible that are executed if the object is in state 2, 3, or 4 respectively. Considering the current state, only one intervention out of the three possible has to be considered for each object in the case study area. Exceptions are the track objects, for which the track renewal has always to be considered due to the structural dependencies with bridges. For example, a track has to be renewed if the underlying bridge is renewed independently on the current state of the track. The tables in Annex B summarise the objects, the possible interventions on them, the costs of the interventions, and the benefit in terms of risk reduction due to the execution of this interventions. Interventions can be executed within different time windows. Most of the possible interventions can be executed within all time windows considered (e.g. night, weekend, weekday). A few of them, however, can only be executed within a specific time window. Each time window is related to different losses in service due to different traffic volumes on the track. Losses in service are related to the closure of one or multiple of the eleven track segments. It is differentiated whether a segment is closed individually or together with the parallel track segment leading to a full closure of the line. The losses in level of service and the user costs are summarised in Table 20. Beside the structural dependencies mentioned above in the context of track renewal interventions, economical and topological dependencies are considered. Economical dependencies are considered for track and switch interventions. They reduce the intervention costs of a track intervention by 20 % and a switch intervention by 40 % when the same intervention is executed on a connected track object or a neighbouring switch, respectively. Two chains of track objects are considered for economic dependencies reducing intervention costs, the objects on the up-line and down-line. The switches build four groups with economic dependencies, one close to each of the four stations. Topological dependencies exist between switches and bridges, and between different bridges, that affect the same track segments. Interventions on both object categories can be executed together and reduce the overall duration of losses in service and, therefore, user costs.

6.2 Approaches to develop an optimal intervention program

The optimal intervention program is determined using three different approaches. First, an exhaustive search is performed to find the optimal intervention program out of all possible ones. The second approach tries to simplify the entire system by stating decision rules. Decision rules are often used to simplify complex systems. They, however, have to be stated considering the exact situation in order to develop the optimal or a near optimal intervention program. The last approach uses the network flow model presented in section 4.3. The objective function and constraints explained in sections 4.1 and 4.2 are used in all three approaches.

6.2.1 Exhaustive search with a reduced search space

An exhaustive search considers all possible intervention programs and finds the optimal one by estimating the net benefit for each of it separately. Considering all combinations of the example would mean to test $3 * 10^{82}$ possible intervention programs. This estimation is shown in Table 23, whereby the number of possibilities per object is estimated by multiplying the number of traffic windows, the number of possible track closures that refers to a single track closure or a complete link closure, and the number of interventions. One is added to this number of possible interventions to count for the situation when no intervention is executed on the object. Lastly, the possibility is put in power of the number of objects to get the combinatorial possibilities of all objects.

Table 23. Total number of possible intervention programs

	Tracks	Switches	Bridges	Total
Number of traffic windows: Number of track closures (single track closure or complete closure): Number of interventions:	3 2 3	3 2 3	3 1 3	
Possibilities per object + No intervention is executed	18 19	18 19	9 10	
Number of objects	11	23	39	
Possibilities	$1*10^{14}$	$3*10^{29}$	$1*10^{39}$	$\mathbf{3 * 10^{82}}$

To test $3 * 10^{82}$ possibilities would take ages. It can, however, be reduced significantly by considering the objects current state. This reduces the solution space of the original combinatorial problem by only considering realistic intervention programs. The first reduction is by reducing the number of objects. All objects, excluding tracks, with a current risk equal to 0 can be omitted from the exhaustive search since they will never be part of an intervention program. Tracks have to be excluded from this due to their structural dependencies from bridge renewal interventions. For example, a track could require a renewal intervention due to a bridge renewal even when the risk related to the track itself is zero. This way, 4 switch and 16 bridge objects can be omitted from the investigation and $2 * 10^{61}$ possible intervention program remain.

The second reduction is achieved by omitting interventions that are executed on other states than the current objects states. This means that only one possible intervention is considered per object except for tracks. There again, a track renewal has to be considered independently of the state due to structural dependencies. 10^{42} possible intervention programs remain.

The possibilities can be further reduced to $4 * 10^{35}$ by the consideration of the intervention durations and therefore the traffic windows possible for the execution. This does not change the numbers from track and switches because track intervention can always be executed in all time windows and due to the switches states only manual grinding is considered, which can also be executed in all time windows. In respect of bridges, all except two have possible interventions that can only be executed during weekdays, which reduces the number of possible traffic windows to one. The other two bridge interventions could be executed within two traffic windows. Table 24 shows the calculation of the possible intervention programs after the reduction.

Table 24. Total number of possible intervention programs after possible reductions

	Tracks	Switches	Bridges		Total
Number of traffic windows:	3	3	1	2	
Number of track closures (single track closure or complete closure):	2	2	1	1	
Number of interventions:	2	1	1	1	
Possibilities per object	12	6	1	2	
+ No intervention is executed	13	7	2	3	
Number of objects	11	19	21	2	
Possibilities	$2*10^{12}$	$1*10^{16}$	9	$2*10^{6}$	**$4 * 10^{35}$**

These $4*10^{35}$ possibilities are still excessively many to be tested in reasonable time. For the situation without a budget limitation, it is possible to divide the problem into sub problems which can be computed much faster and then in a second step being put together again. To name this approach exhaustive search can still be justified with the performance of exhaustive searches for each sub problem. The optimal intervention program of all tracks and of each group of economically dependent switches are developed separately for all $18*10^{6}$ possibilities bridge intervention programs. One of these possible bridge intervention programs is for example the rebuild of bridges B_{16}, B_{28} and B_{38}. Tracks and switches can be split since they do not have any related dependencies. The net benefit of the optimal intervention programs of tracks and economic groups of switches are added to the related intervention program for the bridges, out of which the overall optimal intervention program can be found. For example, the optimal intervention program for all tracks in the situation when bridges B_{16}, B_{28} and B_{38} are subject to rebuild interventions is added to the intervention program for bridges that includes the rebuild of bridges B_{16}, B_{28} and B_{38}. This requires the testing of $2*10^{12}$ intervention programs. The track intervention program with the maximum net benefit is adjusted to the bridge intervention program where no bridge intervention is selected. This divided exhaustive search reduces the number of values that have to be determined to $4.5*10^{12}$, which can be done within reasonable time.

This approach, however, does not work for the situation with a budget limitation. There, it is not possible to divide the overall problem into sub-problems that are later added up since the additional constraint of the budget limitation is over all objects. It is, therefore, not possible to do an exhaustive search to develop the optimal intervention program for the situation with a budget limitation within reasonable time.

6.2.2 Decision rules

Decision rules are introduced in order to reduce the combinatorial complexity of the problem. The aim is to eliminate many possible combinations, which are either not feasible or far from optimal, while still being able to determine a near optimal intervention program. Decision rules are case oriented and may not be suitable in other situations, with other track layout, or interventions considered. An often-used rule in praxis is:

Interventions are always executed with the least possible loss in service.

In this example, this rule is used as a decision rule when developing intervention programs. Table 25 states two case specified rules derived from the general rule and shows their impact on the different object categories and the network dependencies.

Table 25. Rules and their influence on object categories

	Tracks*	Switches	Bridges
1. Whenever possible, interventions are executed during the night.			
Description	Track interventions can be executed at any time, and can therefore be executed during the night.	Manual grinding and welding, both take 3 h per object, and can therefore be executed during the night.	No bridge intervention can be executed during the night.
Economical dependencies	Can be considered. Grouped track interventions can be divided into different night shifts like a single track intervention.	Cannot be considered, because one night shift (4 h) is shorter than the required time for two such interventions (6 h).	N/A
Topological dependencies	N/A	Do not have any influence due to the zero user costs during the night.	
2. An intervention can be executed when the network is in all traffic states with a higher loss in service if, and only if, it can be grouped with another, longer lasting intervention that requires this traffic state.			
Description	Track interventions cannot be executed at the same time as another intervention on the same track segment.	Switch interventions can be executed at the same time as bridge interventions on the same track segment.	Interventions on different bridges affecting the same line can be executed at the same time.
Economical dependencies	N/A	Can be considered, as long as the grouped duration does not lead to higher losses in level of service.	N/A
Topological dependencies	N/A	Switch interventions can make advantage of topological dependencies with bridge interventions and be executed at the same time when the duration of the bridge intervention is longer than the duration of the switch intervention.	Can be considered between bridges affecting the same line. A shorter intervention can be executed in the line closure of a longer intervention.

* Exceptions are mandatory interventions due to structural dependencies, e.g. a track renewal due to a bridge renewal. These have to be grouped with the initial bridge intervention.

Rule one can be justified by the comparison between the reduction in owner costs due to grouping interventions together and the increase in user costs due to the execution during other time windows with longer possible work durations. For example, a switch welding costs 10’000 euros and takes 3 hours, which can be done within a night window. A night window allows a maximum work duration of 4 hours. Considering another switch with a welding intervention on, the owner could save a 4’000 euros (40% of 10’000) when grouped with the first switch. The total duration, however, becomes 6 hours (2 times 3), which requires the interventions to be executed during a weekend instead of a night window. The user costs on weekends, which are zero for night work, are around 2’000 euros per hour (an average value for weekend closures of single track segments according to Table 20). Multiplied by the total duration of 6 hours, this totals in 12’000 euros of additional user costs. The comparison of owner cost savings of 4’000 euros and user cost increases of 12’000 euros justifies this rule.

Rule two enables to execute interventions in traffic states having a higher loss in level of service than the traffic state required from the interventions itself, if, and only if, the interventions can be combined with another intervention that requires this “higher” traffic state anyway. Coming back to the example with the two switch interventions above, it is assumed that there is a bridge renewal planned affecting the same tracks as the two switches do. The bridge renewal takes 72 hours and is executed during weekdays. The switch interventions can then be grouped and executed during the same time as the bridge is renewed because the grouped duration of 6 hours is less than the 72 hours for which the tracks have to be closed due to the bridge intervention.

Table 25 describes the impact of each rule independently. Summarised, the rules define clearly how each intervention has to be dealt with. This is shown in Table 26.

Table 26. Consideration of interventions due to decision rules

Intervention	How to consider	Reasoning
Track Tamping	Are executed during the night except when the track has to be renewed due to structural dependencies. Economical dependencies are considered.	This interventions can be divided anytime and therefore be executed during the night. Two grouped interventions can be seen as one single intervention that lasts longer in the execution, which, however, can then be divided into multiple night again.
Track Ballast Cleaning		
Track Renewal		
Switch Manual Grinding	Are executed during the night without the possibility of grouping. Weekend or Weekday execution is possible to allow grouping when a bridge intervention is executed that lasts longer than the total duration of the grouped interventions	A night window allows 4 hours of work, which is more than the duration of one intervention (3 hours), but less than the duration of two grouped interventions (6 hours). During the weekend or weekdays, longer windows are possible and interventions can be grouped to reduce the intervention costs. It is, however, only possible to execute these interventions during the weekend or weekday when another intervention is executed, which would require this traffic state anyway.
Switch Welding		
Switch Renewal	Are executed during a weekend without the possibility of grouping. Weekday execution is possible to allow grouping when a bridge intervention is executed that lasts longer than the total duration of the grouped interventions	A weekend window allows 52 hours of work, which is more than the duration of one intervention (36 hours), but less than the duration of two grouped interventions (72 hours). During weekdays, no time limitation exist and interventions can be grouped to reduce the intervention costs. It is, however, only possible to execute interventions during weekdays when another intervention is executed then, which would require this traffic state anyway.
Bridge Recoating	Are executed on weekends or weekdays dependently on their duration. Two interventions affecting the same lines are executed parallel in time in the traffic state required by the longer lasting intervention	These interventions can be executed at the same time when affecting the same track routes, which reduces losses in level of service. An intervention usually executed on the weekend can be moved to another intervention executed during weekdays without increasing the duration then due to the fact that an intervention that is usually executed on a weekend is shorter than an intervention executed during weekdays.
Bridge Strengthening		
Bridge Major Repair / Renewal		

This decision rules allow to hierarchically split the problem into sub-problems for each object category and even groups of economic dependent objects, where the bridge interventions are on a higher hierarchical level than tracks and switches. First, bridges can be considered separately from the other object categories since they do not get additional benefits for themselves when grouped with interventions from other object categories. In topological dependencies with switches, bridge interventions are always the dominating intervention that means that the loss in service is higher for the bridge interventions than for the topological dependant switch interventions. The only thing that has to be considered are the mandatory track interventions that may be required due to bridge renewals. Their costs and durations can be included in the cost and duration of bridge renewals.

Second, track interventions are mostly independent of the other object categories. They can always be executed during the night and still consider economic dependencies between interventions. The only relation to the other object categories are the mandatory interventions due to bridge renewals, which are already included in the bridge interventions. A track, therefore, that has already a mandatory renewal intervention due to structural dependencies with bridges cannot have another intervention at the same time and therefore should be excluded in the track consideration.

Third, switches can be considered within their groups of economically dependant objects. Due to the duration of switch interventions, the economic benefit from grouping them cannot be achieved without executing them in a time window with higher loss in level of service than an individual execution would require. This, however, is prohibited by the rules as long as there is not a bridge intervention executed leading to a longer lasting closure of the line. The longer lasting is not a problem in the example situation since the bridge interventions have much longer durations than the switch interventions. The interventions on switches, therefore, can only be executed individually when no bridge intervention is executed on the line. On the other hand, interventions on switches with economic dependencies can be, and are due to the lower owner costs, executed grouped when a bridge intervention is executed within the line.

6.2.3 Network flow model

The network flow model presented in section 4.3 is used. Table 27 shows the characteristics of the network flow model generated within the model. The 73 objects considered lead to 408 possible interventions executed on them, which can be executed within 45 traffic states. Table 28 shows the characteristics of the mixed integer linear program that solves the network flow model. The mixed integer linear program, therefore, consists of 2’774 decision variables and 1’869 constraints.

Table 27. Characteristics of the network flow model in the example

Characteristics	Intervention layer	All 45 traffic state layers	Interlayer	Total
Number of nodes	410	691	-	1101
Number of edges	1156	927	691	2083

Table 28. Characteristics of the mixed integer linear program from the network flow model

Characteristics	Size
Number of decision variables	
- Intervention layer	1156
- Traffic state layers	927
- Interlayer	691
Total:	**2'774**
Number of constraints	
- Flow constraints intervention layer	410
- Flow constraints traffic state layers	736
- Duration transformation constraints	408
- Exclusivity constraints	73
- Equal flow constraints	156
- Organisational constraints	1
- Structural constraints	85
Total:	**1'869**

6.3 Results

Table 29 shows the intervention programs determined using the three approaches for the situation with an unlimited budget. The table lists all objects having an intervention in one of the three intervention programs and neglects the objects that are never included in the intervention programs to improve the comparability of the results. It is shown which intervention is executed on the object and under which traffic state. More detailed tables and a graphical representations are provided in Annex C, Annex D, and Annex E for the exhaustive search, the decision rule approach, and the network flow model, respectively. The costs and benefit, as well as the net benefit of the intervention programs developed by all three approaches are shown in Table 30.

All three intervention programs include the same objects. These are all track and switch objects that are in a not excellent state (states 2,3, and 4) and the two bridges B_{16} and B_{28}. Tracks T_3, T_4, T_5, and T_6 are renewed even though they would not require a renewal due to their state alone, if they would not have to be renewed due to the bridge renewal interventions. This renewal is necessary due to the bridge renewal interventions within their segments. Bridges B_{16} and B_{28} are in poor state and compared with all other bridge objects have a risks associated with them that are bigger than the costs occurring due to their interventions. Switches S_1, S_2, S_3, S_8, S_9, S_{10}, and S_{11} are executed during the day without increasing the user costs due to the renewal of bridge B_{16} and B_{28}, respectively. These switch interventions can be executed parallel to the bridge renewals. Due to the day execution of the grinding interventions on the switches, fixed intervention costs can be shared between these interventions, which would not be possible during the night. The intervention programs developed by exhaustive search and the network flow model include a weekend execution of switches S_{21} and S_{22}, which are executed during the night in the intervention program of the decision rules approach. This is the only difference between the three intervention programs.

This difference is also visible in the costs shown in Table 30. While the benefit of each intervention program is the same, which only depends on the intervention selection, the user and owner costs differ slightly due to the weekend execution of switches S_{21} and S_{22}. The owner and user costs for the intervention program developed by decision rules are 166'000 euros higher and 76'000 euros lower, respectively. Together, the intervention program by decision rules is about 90'000 euros more expensive for the same risk reduction as the other intervention programs provide. The overall net benefit is about 52.2 Mio euros for the intervention program by exhaustive search and the network flow model, and 52.0 Mio euros for the intervention program by decision rules.

Table 29. Intervention programs developed without a budget constraint

ID	Exhaustive search		Decision Rules		Network flow model	
	Intervention	Traffic state	Intervention	Traffic state	Intervention	Traffic state
T_1	Tamping	Night	Tamping	Night	Tamping	Night
T_2	Tamping	Night	Tamping	Night	Tamping	Night
T_3	Track Renewal	T_3, Day	Track Renewal	T_3, Day	Track Renewal	T_3, Day
T_4	Track Renewal	T_4, Day	Track Renewal	T_4, Day	Track Renewal	T_4, Day
T_5	Track Renewal	T_5, Day	Track Renewal	T_5, Day	Track Renewal	T_5, Day
T_6	Track Renewal	T_6, Day	Track Renewal	T_6, Day	Track Renewal	T_6, Day
T_9	Ballast Cleaning	Night	Ballast Cleaning	Night	Ballast Cleaning	Night
T_{10}	Ballast Cleaning	Night	Ballast Cleaning	Night	Ballast Cleaning	Night
T_{11}	Tamping	Night	Tamping	Night	Tamping	Night
S_1	Manual Grinding	T_1, T_2, T_3, T_4, T_{11}, Day	Manual Grinding	T_1, T_2, T_3, T_4, T_{11}, Day	Manual Grinding	T_1, T_2, T_3, T_4, T_{11}, Day
S_2	Manual Grinding	T_1, T_2, T_3, T_4, T_{11}, Day	Manual Grinding	T_1, T_2, T_3, T_4, T_{11}, Day	Manual Grinding	T_1, T_2, T_3, T_4, T_{11}, Day
S_3	Manual Grinding	T_1, T_2, T_3, T_4, T_{11}, Day	Manual Grinding	T_1, T_2, T_3, T_4, T_{11}, Day	Manual Grinding	T_1, T_2, T_3, T_4, T_{11}, Day
S_8	Manual Grinding	T_5, T_6, Day	Manual Grinding	T_5, T_6, Day	Manual Grinding	T_5, T_6, Day
S_9	Manual Grinding	T_5, T_6, Day	Manual Grinding	T_5, T_6, Day	Manual Grinding	T_5, T_6, Day
S_{10}	Manual Grinding	T_5, T_6, Day	Manual Grinding	T_5, T_6, Day	Manual Grinding	T_5, T_6, Day
S_{11}	Manual Grinding	T_5, T_6, Day	Manual Grinding	T_5, T_6, Day	Manual Grinding	T_5, T_6, Day
S_{12}	Manual Grinding	Night	Manual Grinding	Night	Manual Grinding	Night
S_{13}	Manual Grinding	Night	Manual Grinding	Night	Manual Grinding	Night
S_{14}	Manual Grinding	Night	Manual Grinding	Night	Manual Grinding	Night
S_{15}	Manual Grinding	Night	Manual Grinding	Night	Manual Grinding	Night
S_{16}	Manual Grinding	Night	Manual Grinding	Night	Manual Grinding	Night
S_{17}	Manual Grinding	Night	Manual Grinding	Night	Manual Grinding	Night
S_{18}	Manual Grinding	Night	Manual Grinding	Night	Manual Grinding	Night
S_{19}	Manual Grinding	Night	Manual Grinding	Night	Manual Grinding	Night
S_{20}	Manual Grinding	Night	Manual Grinding	Night	Manual Grinding	Night
S_{21}	Manual Grinding	**T_{10}, Weekend**	Manual Grinding	**Night**	Manual Grinding	**T_{10}, Weekend**
S_{22}	Manual Grinding	**T_9, Weekend**	Manual Grinding	**Night**	Manual Grinding	**T_9, Weekend**
S_{23}	Manual Grinding	Night	Manual Grinding	Night	Manual Grinding	Night
B_{16}	Renewal	T_1, T_2, T_3, T_4, T_{11}, Day	Renewal	T_1, T_2, T_3, T_4, T_{11}, Day	Renewal	T_1, T_2, T_3, T_4, T_{11}, Day
B_{28}	Renewal	T_5, T_6, Day	Renewal	T_5, T_6, Day	Renewal	T_5, T_6, Day

Table 30. Net benefit of the intervention programs developed without a budget constraint

Cost element	Exhaustive search	Decision Rules	Network flow model
Owner costs: - Fix intervention costs - Variable intervention costs Total:	 1'617'133 7'022'308 8'639'441	 1'783'077 7'022'308 8'805'385	 1'617'133 7'022'308 8'639'441
User costs:	6'172'874	6'097'064	6'172'874
Total costs	14'812'315	14'902'449	14'812'315
Benefit in terms of risk reduction	66'927'606	66'927'606	66'927'606
Net benefit	**52'188'462**	**52'025'157**	**52'188'462**
Percentage of maximal net benefit	100 %	99,7 %	100 %

Table 31 and Table 32 show the results for the situation with a budget limitation of 4 Mio euros. The intervention programs of the unlimited situation with user costs in the range of 8.6-8.8 Mio euros cannot be executed anymore. While the intervention program developed by decision rules and the network flow model differ in the unlimited situation, they are equal in the budget limited case. The intervention programs do not include any switch intervention. They include the renewal of bridge B_{16} due to the large risks associated with it that can be reduced and track interventions on tracks T_1, T_2, T_3, T_4, T_9, and T_{11}. Tracks T_3 and T_4 are renewed during the day due to the renewal intervention on bridge B_{16}. These three interventions together cost with 3'994'800 euros that is almost up to the 4 Mio euros of the constraint, i.e. 3.2 Mio euros for the bridge renewal and 397'400 for each track renewal. Only 5'200 euros can be spent on other interventions. Therefore, the inexpensive interventions on tracks T_1, T_2, T_9, and T_{11} are included instead of any switch intervention.

The net benefit of this intervention program is about 3.9 Mio euros, which is only 7.5 % of the net benefit generated by the optimal intervention program when no budget limitation exist.

Table 31. Intervention programs developed with a budget limitation of 4 Mio euros

ID	Exhaustive search		Decision Rules		Network flow model	
	Intervention	Traffic state	Intervention	Traffic state	Intervention	Traffic state
T_1	N/A		Tamping	Night	Tamping	Night
T_2			Tamping	Night	Tamping	Night
T_3			Track Renewal	T_3, Day	Track Renewal	T_3, Day
T_4			Track Renewal	T_4, Day	Track Renewal	T_4, Day
T_9			Ballast Cleaning	Night	Ballast Cleaning	Night
T_{11}			Tamping	Night	Tamping	Night
B_{16}			Renewal	T_1, T_2, T_3, T_4, T_{11}, Day	Renewal	T_1, T_2, T_3, T_4, T_{11}, Day

Table 32. Net benefit of the intervention programs developed with a budget limitation of 4 Mio euros

Cost element	Exhaustive search	Decision Rules	Network flow model
Owner costs: - Fix intervention costs - Variable intervention costs Total:	N/A	 799'986 3'199'946 3'999'932	 799'986 3'199'946 3'999'932
User costs:		3'045'135	3'045'135
Total costs		7'045'067	7'045'067
Benefit in terms of risk reduction		10'912'647	10'912'647
Net benefit		**3'867'580**	**3'867'580**
Percentage of maximal net benefit		100 %	100 %

Additionally to the intervention programs and the costs and benefit, Table 33 lists all traffic states with their duration and related user costs that are required to execute the intervention program developed by the network flow model. All traffic states regarding the night have been consolidated since they never cause user costs. The values count also the intervention programs developed with exhaustive search when there is no budget limitation and with decision rules when there is a budget limitation, respectively, because they are the same intervention programs as the ones developed by the network flow model.

The duration column shows the total duration a traffic state is required. This could be due to one intervention or due to multiple interventions not requiring the entire duration to be on one block. It can be seen that the complete closure of a line (where more than one track segment is closed) share the most of the total user costs.

Table 33. Traffic states of the intervention program for the network flow model

Closed track segments	Time window	Without budget limitation		With budget limitation	
		Duration [h]	Costs [euro]	Duration [h]	Costs [euro]
T3	Day	4.5	22'429	4.5	22'429
T4	Day	4.5	22'429	4.5	22'429
T5	Day	4.6	31'570	-	-
T6	Day	4.6	31'570	-	-
T1, T2, T3, T4, T11	Day	72.0	3'000'276	72.0	3'000'276
T5, T6	Day	72.0	2'988'789	-	-
T9	Weekend	3.0	0	-	-
T10	Weekend	3.0	2'639	-	-
Different track segments	Night	35.8	0	5.6	0
Total user costs:			**6'097'063**		**3'045'135**

6.4 Discussion

Regarding the results of the unlimited budget situation shown in the former section, it can be seen that the network flow model develops the same intervention program as the exhaustive search, which is the optimal intervention program. The net benefit of the optimal intervention program amount to 52.2 Mio euros consisting of a benefit in terms of risk reduction of 66.9 Mio euros, owner costs of 8.6 Mio euros, and user costs of 6.2 Mio euros. The benefit is mostly driven by the two renewal interventions on bridges B_{16} and B_{28}. Even though, their 3.2 and 3.8 Mio euros of intervention costs are much higher than the intervention costs for the tack and switch interventions, their risk reduction of about 10 Mio euros and 54 Mio euros, respectively, represent almost the entire share of benefit in the intervention program. Their owner costs are about 81% of the entire owner costs of the intervention program, while the benefit share lays at 96%.

The intervention program developed by decision rules includes the same interventions in the intervention program as the optimal intervention program, but attributes other time windows to the execution of the intervention on switches S_{21} and S_{22}. While they are executed over the weekend in the optimal intervention program, the decision rule does not include this option at all. Due to the rule that an intervention that can be executed during the night has to be executed during the night, where the loss in level of service is equal to zero, and that a deviation from this rule is only permissible in the case of a simultaneously executed bridge intervention, the option to execute two switch interventions during the weekend instead of during the night without having a bridge intervention at the same time is not considered. This weekend execution belongs to the optimal intervention program even though it leads to losses in service, which a night execution would not. The losses in service, though, are still lower than the reduction in intervention costs due to the possibility of grouping the two switch interventions. For example, the closure of the affected tracks during the weekend costs the users 2'640 euros. The fixed costs of the interventions that are saved due to the grouped execution is 4'000 euros. The weekend execution for these two switches is, therefore, better. For all other switches, this would not be better due to the much higher user costs that are related to a closure of the adjusted tracks. This is an example of how the simplification due to clear stated decision rules reduces the possible combinations and with it the solution spaces which could include the omission of the optimal intervention program.

Regarding the situation with a budget limitation, the intervention programs developed by decision rules and the network flow model are the same and it is assumed that this is the optimal intervention program for this situation. The net benefit is 3.9 Mio euros, which is only 7.5% of the net benefit of the optimal intervention program without a budget limitation. This huge difference in net benefit is due to the high benefit in terms of risk reduction achieved by executing a renewal intervention on bridge B_{28}, which is not included in the intervention program with limited budget. It is not included because the intervention costs of this intervention and the costs of the mandatory track renewals on T_3 and T_4 are already above the budget limitation. For example, the bridge renewal costs 3.8 Mio euros and each track intervention that are required due to the renewal of the bridge costs 405'000 euros. All three together costs more than 4.6 Mio euros, which is already 0.6 Mio euros above the budget limitation. A budget limitation of 4 Mio euros, therefore, excludes 54.5 Mio euros of risk reduction.

The results in section 6.3 show that the network flow model determines the optimal intervention program for the example situation while the decision rule approach is close to the optimal too. These two approaches enable a faster computation of the intervention programs either due to the formulation of a constrained network flow model or due to the reduction in combinatorial complexity by stating straightforward rules. Table 34 shows the computational effort needed to develop the intervention program. It can be seen that both approaches require much less computational effort than the exhaustive search. Further, they have been able to find an optimal solution within under one minute for the situation with a budget limitation, what would have required the exhaustive search to run approximately for 10^{12} years. The difference between the network flow model and the decision rule approach is due to the reduced considerations in the latter.

Table 34. Computation time for the different models

Model	Without budget limitation	With budget limitation
Exhaustive search	139 min	N/A
Decision rule	1 s	2 s
Network flow model	16 s	38s

7 Summary and Conclusions

In this report, a methodology is presented to develop optimal intervention programs for railway infrastructure network. The railway infrastructure network is analysed with respect to the important characterises in terms of intervention planning. Based on the analysed characteristics, a conceptual model is presented that maximises the net benefit considering owner costs, user costs, and organisational constraints, i.e. budget limitation. This general model formulation allows consideration of benefit in terms of the risk reduced through the execution of interventions. The risk reduction is estimated based on the risk assessment process developed in task 3.2 of the DESTination Rail project and it is shown how this can be included in the development of intervention programs for railway infrastructure networks (Papathanasiou et al., 2016). Further, the model allows consideration of different dependencies within the network, including economical dependencies between similar interventions allowing to reduce intervention costs, structural dependencies enabling the consideration of the relation between interventions where one intervention requires the execution of the second intervention, and topological dependencies that allow consideration of the relation between objects in terms of the system functionality.

In the report, a network flow model is presented that allows to model the development of optimal intervention programs for railway infrastructure network as a constrained multilayer minimum cost flow problem. This model is based on a minimum cost flow problem that searches for the flow with the minimum cost within a given network under the consideration of flow conservation in each node. A constrained and generalised form allows to consider additional constraints, i.e. a budget constraint, and gains and losses within the network. The model is built as a multilayer network, where one layer represents the intervention selection and each of the other layers one particular traffic state determine the way how the traffic network is used.

This model is applied on an example network with 73 objects, i.e. 11 tracks, 23 switches, and 39 bridges. There, optimal intervention programs are developed for a situation without a budget limitation and a situation with a limited budget. It is shown that the network flow model is able to develop the optimal intervention program in much faster computational time than the exhaustive search would need. Additionally, a more simplified approach based on straightforward decision rules is applied that is able to find a near optimal intervention program in even less time. This approach reduces the possible combinations by omitting many solutions due to the rules put in place. The example shows the benefit and limitation of such an approach. It takes even less time to compute the intervention program with this approach, which, however, is not always able to develop the optimal intervention program due to the omission of many possible intervention programs.

8 References

Aksentijevic, J., Blieberger, J., Stefan, M., & Schöbel, A. (2015). *List of common problems faced by rail infrastructure. DESTination RAIL Deliverable D1.1*. Vienna, Austria. Retrieved from http://www.destinationrail.eu/documents

Aksentijevic, J., Blieberger, J., Stefan, M., & Schöbel, A. (2017). *Report on Traffic Flow Model. DESTination RAIL Deliverable D4.2*. Vienna, Austria.

Andrade, A. R., & Teixeira, P. F. (2012). Biobjective Optimization Model for Maintenance and Renewal Decisions Related to Rail Track Geometry. *Transportation Research Record: Journal of the Transportation Research Board*, *2261*(1), 163–170. https://doi.org/10.3141/2261-19

Bertsekas, D. P. (1998). *Network Optimization: Continuous and Discrete Models*. Athena Scientific, Belmont, Massachusetts. https://doi.org/10.1.1.86.448

Budai-Balke, G. (2009). *Operations research models for scheduling railway infrastructure maintenance*. Erasmus University, Rotterdam.

Budai, G., Huisman, D., & Dekker, R. (2004). Scheduling preventive railway maintenance activities. *2004 IEEE International Conference on Systems, Man and Cybernetics (IEEE Cat. No.04CH37583)*, *5*, 4171–4176. https://doi.org/10.1057/palgrave.jors.2602085

Budai, G., Huisman, D., & Dekker, R. (2006). Scheduling preventive railway maintenance activities. *Journal of the Operational Research Society*, *57*, 1035–1044. https://doi.org/10.1057/palgrave.jors.2602085

Caetano, L. F., & Teixeira, P. F. (2013). Availability Approach to Optimizing Railway Track Renewal Operations. *Journal of Transportation Engineering*, *139*(9), 941–948. https://doi.org/10.1061/(ASCE)TE.1943-5436.0000575

Caetano, L. F., & Teixeira, P. F. (2014). Optimisation model to schedule railway track renewal operations: a life-cycle cost approach. *Structure and Infrastructure Engineering*, *2479*(August), 1–13. https://doi.org/10.1080/15732479.2014.982133

Chen, S. kuan, Ho, T. kin, & Mao, B. hua. (2013). Maintenance schedule optimisation for a railway power supply system. *International Journal of Production Research*, *51*(16), 4896–4910. https://doi.org/10.1080/00207543.2013.774501

Dekker, R., Wildeman, R. E., & van der Duyn Schouten, F. A. (1997). A review of multi-component maintenance models with economic dependence. *Mathematical Methods of Operations Research*, *45*(3), 411–435. https://doi.org/10.1007/BF01194788

Den Hertog, D., Van Zante-De Fokkert, J. I., Sjamaar, S. A., & Beusmans, R. (2005). Optimal working zone division for safe track maintenance in the Netherlands. *Accident Analysis and Prevention*, *37*(5), 890–893. https://doi.org/10.1016/j.aap.2005.04.006

DESTination RAIL. (2018). *Whole Life Cycle Analysis. DESTination RAIL Deliverable D4.3*. Retrieved from http://www.destinationrail.eu/documents

Eicher, C., Lethanh, N., & Adey, B. T. (2015). A routing algorithm to construct candidate workzones with distance constraints. *Proc., ICSC15: The Canadian Society for Civil Engineering 5th International / 11th Construction Specialty Conference*. Vancouver, Canada: University of British Columbia. https://doi.org/10.14288/1.0076440

Esveld, C. (2014). *Modern Railway Track*.

European Union. (2012). Directive 2012/34/EU Establishing a Single European Railway Area. *Official Journal of the European Union*, *L343*(32). Retrieved from http://eur-lex.europa.eu/legal-content/EN/ALL/?uri=CELEX:32012L0034&qid=1480601329917

Fecarotti, C., & Andrews, J. D. (2017). *Optimising strategy selection for the management of railway assets*. The University of Nottingham, United Kingdom.

Fendrich, L., & Fengler, W. (2013). *Handbuch Eisenbahninfrastruktur*. Springer-Verlag.

Frangopol, D. M., & Liu, M. (2007). Maintenance and management of civil infrastructure based on condition, safety, optimization, and life-cycle cost. *Structure and Infrastructure Engineering*, *3*(1), 29–41. https://doi.org/10.1080/15732470500253164

Freystein, H., Muncke, M., & Schollmeier, P. (2015). *Handbuch Entwerfen von Bahnanlagen* (3.). DVV Media Group GmbH, Hamburg.

Furuya, A., & Madanat, S. M. (2013). Accounting for Network Effects in Railway Asset Management. *Journal of Transportation Engineering*, *139*(January), 120822055938002. https://doi.org/10.1061/(ASCE)TE.1943-5436.0000477

Guler, H. (2013). Decision support system for railway track maintenance and renewal management. *Journal of Computing in Civil Engineering*, *27*(3), 292–306. https://doi.org/10.1061/(ASCE)CP.1943-5487.0000221

Gutsche, K. (2009). Integrierte Bewertung von Investitions- und Instandhaltungsstrategien für die Bahnsicherungstechnik.

Hajdin, R., & Adey, B. T. (2006). Optimal worksites on highway networks subject to constraints. Zurich, Switzerland: International Forum on Engineering Decision Making.

Higgins, A., Ferreira, L., & Lake, M. (1999). Scheduling Rail Track Maintenance to Minimise Overall Delays. *Transportation and Traffic Theory*, 779–796. Retrieved from http://eprints.qut.edu.au/2417/

Jänsch, E. (2016). *Das System Bahn* (2.). DVV Media Group GmbH, Hamburg.

Jenema, A. R. (2011). *An optimization model for a Train-Free-Period planning for ProRail based on the maintenance needs of the Dutch railway infrastructure*. Delft University of Technology Responsible.

Jungnickel, D. (2013). *Graph, Networks, and Algorithms*. (M. Bronstein, A. M. Cohen, H. Cohen, D. Eisenbud, & B. Sturmfels, Eds.), *Algorithms and Computation inMathematics* (4th ed.). Springer.

Kerwin, S., & Adey, B. T. (2017). Determination of optimal intervention programs for a water distribution network. In *The Leading Edge Sustainable Asset Management of Water and Wastewater Infrastructure Conference (LESAM)*. Trondheim, Norway.

Kielhauser, C., & Adey, B. T. (2017). A methodology to determine optimal intervention programs for multiple urban infrastructure networks with a consistent representation of service levels. In *2nd International Symposium on Infrastrucute Asset Management - SIAM 2017* (pp. 1–17). Zurich, Switzerland.

Kielhauser, C., Adey, B. T., & Lethanh, N. (2016). Investigation of a static and a dynamic neighbourhood methodology to develop work programs for multiple close municipal infrastructure networks. *Structure and Infrastructure Engineering*, *2479*, 1–29. https://doi.org/10.1080/15732479.2016.1162818

Kielhauser, C., Martani, C., & Adey, B. T. (2017). Development of intervention programs for inland waterway networks using genetic algorithms. *Structure and Infrastructure Engineering*, *2479*, 1–14. https://doi.org/10.1080/15732479.2017.1373299

Kiessling, F., Puschmann, R., Schmieder, A., & Schneider, E. (2009). *Contact Lines for Electric Railways*. (S. Aktiengesellschaft, Ed.) (2nd ed.). Publicis Publishing, Erlangen.

Lethanh, N., & Adey, B. T. (2016). A real option approach to determine optimal intervention windows for multi-national rail corridors. *Civil Engineering and Management*, *22*(1), 38–46.

Lethanh, N., Adey, B. T., & Burkhalter, M. (2018). Determining an Optimal Set of Work Zones on Large Infrastructure Networks in a GIS Framework, *24*(1), 1–16. https://doi.org/10.1061/(ASCE)IS.1943-555X.0000410.

Lethanh, N., Adey, B. T., & Sigrist, M. (2014). A Mixed-Integer Linear Model for Determining Optimal Work Zones on a Road Network. *International Conference on Engineering and Applied Sciences Optimization*. Athens, Greece: National Technical University of Athens.

Li, D. (2013). A new maintenance model for high-speed railway signaling system. *IEEE ICIRT 2013 - Proceedings: IEEE International Conference on Intelligent Rail Transportation*, 239–244. https://doi.org/10.1109/ICIRT.2013.6696300

Liu, M., & Frangopol, D. M. (2004). Optimal bridge maintenance planning based on probabilistic performance prediction. *Engineering Structures*, *26*(7), 991–1002. https://doi.org/10.1016/j.engstruct.2004.03.003

Lounis, Z. (2006). Risk-based maintenance optimization of aging highway bridge decks. In *Advances in engineering structures, mechanics & construction* (pp. 723–734). Springer.

Lyngby, N., Hokstad, P., & Vatn, J. (2008). RAMS Management of Railway Tracks. In *Handbook of Performability Engineering* (pp. 1123–1145). https://doi.org/10.1007/978-1-84800-131-2

Mehlhorn, G. (2010). *Handbuch Brücken*. Springer, Berlin, Heidelberg.

Morant, A., Larsson-Kraik, P.-O., & Kumar, U. (2014). Data-driven model for maintenance decision support: A case study of railway signalling systems. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, *0*(0), 1–15. https://doi.org/10.1177/0954409714533680

Moreu, F., Spencer, B. F., Foutch, D. A., & Scola, S. (2016). Consequence-based management of railroad bridge networks. *Structure and Infrastructure Engineering*, *13*(April), 273–286. https://doi.org/10.1680/tfitsi.61279.687

National Transport Authority. (2016). *National Heavy Rail Census Report 2016*. Dublin.

Olde Keizer, M. C. A., Flapper, S. D. P., & Teunter, R. H. (2017). Condition-based maintenance policies for systems with multiple dependent components: a review. *European Journal of Operational Research*, *261*(2), 405–420. https://doi.org/10.1016/j.ejor.2017.02.044

OpenStreetMap Contributors. (2017). Open street map. Retrieved from http://www.openstreetmap.org

Papathanasiou, N., Adey, B. T., & Burkhalter, M. (2018). *Risk assessment methodology. DESTination RAIL Deliverable D3.6*. Zurich, Switzerland. Retrieved from http://www.destinationrail.eu/document

Papathanasiou, N., Martani, C., & Adey, B. T. (2016). *Risk Assessment Methodology. DESTination RAIL Deliverable D3.2*. Zurich, Switzerland. Retrieved from http://www.destinationrail.eu/documents

Pargar, F. (2015). A mathematical model for scheduling preventive maintenance and renewal projects of infrastructures. In L. Podofillini, B. Sudret, B. Stojadinovic, E. Zio, & W. Kröger (Eds.), *Safety and Reliability of Complex Engineered Systems* (pp. 993–1000). London: Taylor & Francis Group. https://doi.org/10.1201/b19094-132

Peng, F. (2011, May 25). *Scheduling of track inspection and maintenance activities in railroad networks*. University of Illinois at Urbana-Champaign. Retrieved from https://www.ideals.illinois.edu/handle/2142/24517

Pouryousef, H., Teixeira, P. F., & Sussman, J. (2010). Track Maintenance Scheduling and its Interactions with Operations: Dedicated and Mixed High-speed Rail (HSR) Scenarios. In *2010 Joint Rail Conference. American Society of Mechanical Engineers*. Urbana, IL, USA.

Power, C., Mian, J., Spink, T., Abbott, S., & Edwards, M. (2016). Development of an Evidence-based Geotechnical Asset Management Policy for Network Rail, Great Britain. *Procedia Engineering*, *143*(Advances in Transportation Geotechnics III), 726–733. https://doi.org/10.1016/j.proeng.2016.06.112

Profillidis, V. A. (2014). *Railway Management and Engineering* (4th ed.). Surrey, United Kingdom: Ashgate Publishing Company.

SBB Infrastruktur. (2016). Bahninfrastruktur: Unterhalt und Finanzierung kurz erklärt. Retrieved from https://www.sbb.ch/files/infrastruktur/sbb_themenlandschaft/files/SBBInfraBroschuere_de.pdf

SBB Infrastruktur. (2017). Annual Summary Report 2016, (March).

SMARTRAIL. (2014a). *Deliverable 3.2 Rehabilitation of Open Tracks and Transition Zones*.

SMARTRAIL. (2014b). *Deliverable 3.3 Rehabilitation of Bridges and Tunnels*.

Subramanian, A., Brandstädt, A., & Nishizeki, T. (2016). *Handbook of Graph Theory, Combinatorial Optimization, and Algorithms*. (S. Sahni, Ed.). CRC Press.

Van Horenbeek, A., & Pintelon, L. (2013). A dynamic predictive maintenance policy for complex multi-component systems. *Reliability Engineering and System Safety*, *120*, 39–50. https://doi.org/10.1016/j.ress.2013.02.029

Van Zante-De Fokkert, J. I., Den Hertog, D., Van Den Berg, F. J., & Verhoeven, J. H. M. (2007). The Netherlands schedules track maintenance to improve track workers' safety. *Interfaces*, *37*(2), 133–142. https://doi.org/10.1287/inte.1060.0246

Yianni, P. C., Rama, D., Neves, L. C., Andrews, J. D., & Castlo, D. (2016). A Petri-Net Based Modelling Approach to Railway Bridge Asset Management. *Structure and Infrastructure Engineering*, 0–9.

Zhao, J., Chan, A. H. C., & Burrow, M. P. N. (2009). A genetic-algorithm-based approach for scheduling the renewal of railway track components. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, *223*(6), 533–541. https://doi.org/10.1243/09544097JRRT273

Zhao, J., Chan, A. H. C., Stirling, A., & Madelin, K. (2006). Optimizing Policies of Railway Ballast Tamping and Renewal. *Transportation Research Record: Journal of the Transportation Research Board*, *1943*(1943), 50–56. https://doi.org/10.3141/1943-07

Zwanenburg, W.-J. (2009). *Modeling degradation processes of switches and crossings for* *maintenance and renewal planning on the Swiss railway network. 6th Swiss Transport Research Conference - STRC 06*. École Polytechnique Fédérale de Lausanne.

Annex A Example area

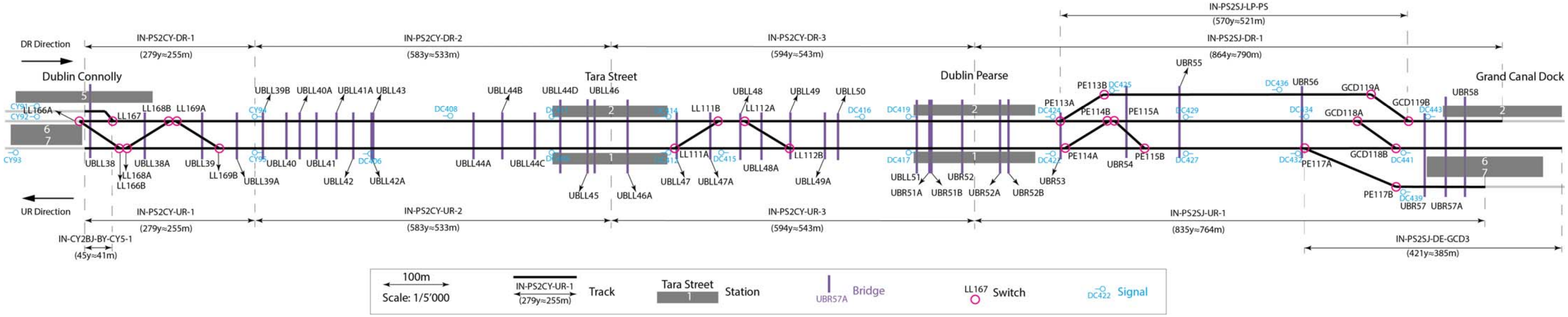

Figure 13. Objects in the example

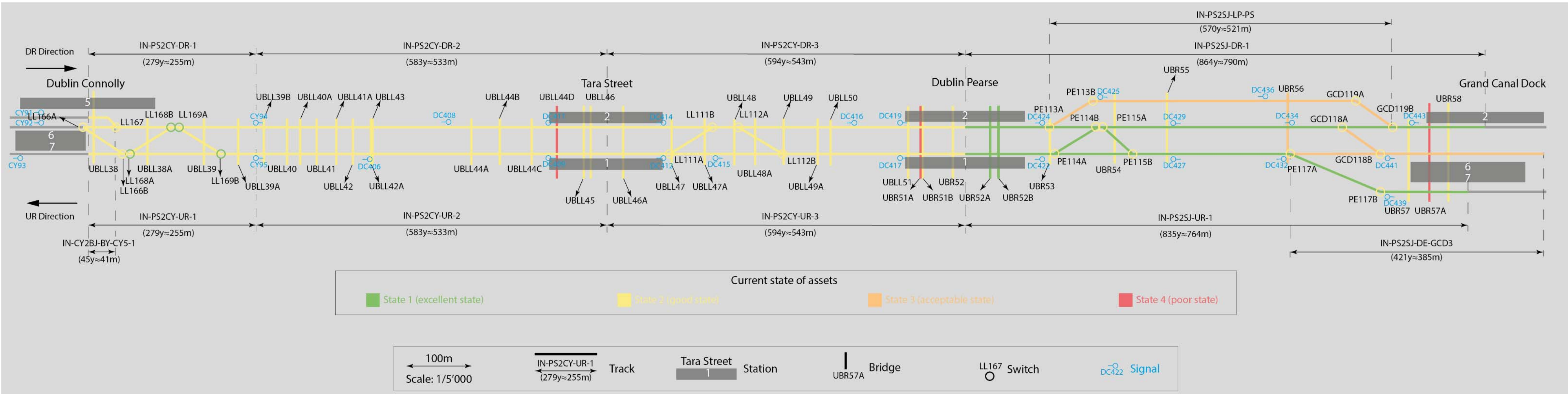

Figure 14. Current state of the objects

Annex B Summarising object table

Table 35. Summary of track objects in the example

ID	Serial number	Dimension [m]	Current state	Possible intervention	Intervention cost [euro]	Risk reduction	Intervention duration [h]	Possible time window		Cost for track renewal [euro]	Duration of track renewal [h]
T_1	IN-PS2CY-DR-1	255	2	Tamping	1'913	56'492	0.56	All		190'128	2.14
T_2	IN-PS2CY-UR-1	255	2	Tamping	1'913	56'492	0.56	All		190'128	2.14
T_3	IN-PS2CY-DR-2	533	2	Tamping	3'998	57'616	1.17	All		397'405	4.48
T_4	IN-PS2CY-UR-2	533	2	Tamping	3'998	57'616	1.17	All		397'405	4.48
T_5	IN-PS2CY-DR-3	543	2	Tamping	4'073	58'402	1.19	All		404'861	4.56
T_6	IN-PS2CY-UR-3	543	2	Tamping	4'073	58'402	1.19	All		404'861	4.56
T_7	IN-PS2SJ-DR-1	790	1	Non	0	0	-	-		589'024	6.64
T_8	IN-PS2SJ-UR-1	764	1	Non	0	0	-	-		569'638	6.42
T_9	IN-PS2SJ-LP-PS	521	3	Ballast Cleaning	990	180'684	4.38	All		388'458	4.38
T_{10}	IN-PS2SJ-DE-GCD	385	3	Ballast Cleaning	732	33'104	3.24	All		287'056	3.24
T_{11}	IN-CY2BJ-BY-CY5-1	41	2	Tamping	308	4'551	0.09	All		30'570	0.34

Table 36. Summary of switch objects in the example

ID	Serial number	Current state	Possible intervention	Intervention cost [euro]	Risk reduction	Intervention duration [h]	Possible time window
S_1	LL166A	2	Manual Grinding	10’000	92'472	3	All
S_2	LL166B	2	Manual Grinding	10’000	92'472	3	All
S_3	LL167	2	Manual Grinding	10’000	92'472	3	All
S_4	LL168A	1	Non	-	0	-	-
S_5	LL168B	1	Non	-	0	-	-
S_6	LL169A	1	Non	-	0	-	-
S_7	LL169B	1	Non	-	0	-	-
S_8	LL111A	2	Manual Grinding	10’000	71'890	3	All
S_9	LL111B	2	Manual Grinding	10’000	71'890	3	All
S_{10}	LL112A	2	Manual Grinding	10’000	71'890	3	All
S_{11}	LL112B	2	Manual Grinding	10’000	71'890	3	All
S_{12}	PE113A	2	Manual Grinding	10’000	68'662	3	All
S_{13}	PE113B	2	Manual Grinding	10’000	60'159	3	All
S_{14}	PE114A	2	Manual Grinding	10’000	67'940	3	All
S_{15}	PE114B	2	Manual Grinding	10’000	67'940	3	All
S_{16}	PE115A	2	Manual Grinding	10’000	68'662	3	All
S_{17}	PE115B	2	Manual Grinding	10’000	68'662	3	All
S_{18}	PE117A	2	Manual Grinding	10’000	67'940	3	All
S_{19}	PE117B	2	Manual Grinding	10’000	68'662	3	All
S_{20}	GCD118A	2	Manual Grinding	10’000	68'662	3	All
S_{21}	GCD118B	2	Manual Grinding	10’000	64'627	3	All
S_{22}	GCD119A	2	Manual Grinding	10’000	60'159	3	All
S_{23}	GCD119B	2	Manual Grinding	10’000	68'301	3	All

Table 37. Summary of bridge objects in the example (part 1)

ID	Serial number	Dimension [m²]	Construction type	Current state	Possible intervention	Intervention cost [euro]	Risk reduction	Intervention duration [h]	Possible time window
B_1	UBLL38	720	Masonry	2	Recoating	180'000	27'340	192.00	Weekday
B_2	UBLL38A	1130	Concrete	2	Recoating	282'500	0	301.33	Weekday
B_3	UBLL39	470	Masonry	2	Recoating	117'500	19'428	125.33	Weekday
B_4	UBLL39A	320	Steel	2	Recoating	80'000	14'865	85.33	Weekday
B_5	UBLL39B	372	Steel	2	Recoating	93'100	0	99.31	Weekday
B_6	UBLL40	167	Steel	2	Recoating	41'650	11'077	44.43	Weekday, Weekend
B_7	UBLL40A	167	Masonry	2	Recoating	41'650	0	44.43	Weekday, Weekend
B_8	UBLL41	350	Steel	2	Recoating	87'500	17'622	93.33	Weekday
B_9	UBLL41A	500	Steel	2	Recoating	125'000	22'975	133.33	Weekday
B_{10}	UBLL42	250	Masonry	2	Recoating	62'500	14'053	66.67	Weekday
B_{11}	UBLL42A	350	Steel	2	Recoating	87'500	0	93.33	Weekday
B_{12}	UBLL43	1410	Masonry	2	Recoating	352'500	55'449	376.00	Weekday
B_{13}	UBLL44A	500	Steel	2	Recoating	125'000	22'975	133.33	Weekday
B_{14}	UBLL44B	450	Masonry	2	Recoating	112'500	21'190	120.00	Weekday
B_{15}	UBLL44C	400	Steel	2	Recoating	100'000	19'406	106.67	Weekday
B_{16}	UBLL44D	640	Steel	4	Renewal	3'200'000	10'499'196	72.00	Weekday
B_{17}	UBLL45	230	Steel	2	Recoating	57'500	13'605	61.33	Weekday
B_{18}	UBLL46	230	Masonry	2	Recoating	57'500	13'605	61.33	Weekday
B_{19}	UBLL46A	960	Steel	2	Recoating	240'000	0	256.00	Weekday
B_{20}	UBLL47	320	Steel	2	Recoating	80'000	16'509	85.33	Weekday
B_{21}	UBLL47A	600	Masonry	2	Recoating	150'000	0	160.00	Weekday
B_{22}	UBLL48	330	Masonry	2	Recoating	82'500	16'865	88.00	Weekday
B_{23}	UBLL48A	460	Masonry	2	Recoating	115'000	0	122.67	Weekday
B_{24}	UBLL49	450	Steel	2	Recoating	112'500	21'132	120.00	Weekday
B_{25}	UBLL49A	650	Steel	2	Recoating	162'500	0	173.33	Weekday

Table 38. Summary of bridge objects in the example (part 2)

ID	Serial number	Dimension [m²]	Construction type	Current state	Possible intervention	Intervention cost [euro]	Risk reduction	Intervention duration [h]	Possible time window
B_{26}	UBLL50	720	Steel	2	Recoating	180'000	30'732	192.00	Weekday
B_{27}	UBLL51	270	Steel	2	Recoating	67'500	14'997	72.00	Weekday
B_{28}	UBR51A	765	Steel	4	Renewal	3’825’000	54'499'698	72.00	Weekday
B_{29}	UBR51B	192	Steel	2	Recoating	48'000	7'448	51.20	Weekday, Weekend
B_{30}	UBR52	110	Steel	2	Recoating	27'500	0	29.33	Weekday, Weekend
B_{31}	UBR52A	160	Masonry	1	Non	-	-	-	-
B_{32}	UBR52B	240	Concrete	1	Non	-	-	-	-
B_{33}	UBR53	345	Masonry	2	Recoating	86'250	0	92.00	Weekday
B_{34}	UBR54	345	Masonry	2	Recoating	86'250	0	92.00	Weekday
B_{35}	UBR55	136	Masonry	2	Recoating	34'000	0	36.27	Weekday, Weekend
B_{36}	UBR56	425	Masonry	3	Strengthening	425’000	13'597	213.00	Weekday
B_{37}	UBR57	187	Masonry	2	Recoating	46'750	0	49.87	Weekday, Weekend
B_{38}	UBR57A	187	Concrete	4	Renewal	1’402’500	3'039'681	72.00	Weekday
B_{39}	UBR58	255	Masonry	2	Recoating	63'750	0	68.00	Weekday

Annex C Results of exhaustive search

Table 39. Intervention program developed by exhaustive search without a budget limitation

ID	Intervention	Traffic state	Duration [h]	Owner costs [Euros]	Risk reduction [Euros]
T_1	Tamping	Night	0.6	1'913	56'492
T_2	Tamping	Night	0.6	1'913	56'492
T_3	Track Renewal	T_3 closed, Day	4.5	397'405	57'616
T_4	Track Renewal	T_4 closed, Day	4.5	397'405	57'616
T_5	Track Renewal	T_5 closed, Day	4.6	404'861	58'402
T_6	Track Renewal	T_6 closed, Day	4.6	404'861	58'402
T_9	Ballast Cleaning	Night	4.4	990	180'684
T_{10}	Ballast Cleaning	Night	3.2	732	33'104
T_{11}	Tamping	Night	0.1	308	4'551
S_1	Manual Grinding	T_1, T_2, T_3, T_4, T_{11} closed, Day	3.0	10'000	92'472
S_2	Manual Grinding	T_1, T_2, T_3, T_4, T_{11} closed, Day	3.0	6’000	92'472
S_3	Manual Grinding	T_1, T_2, T_3, T_4, T_{11} closed, Day	3.0	6’000	92'472
S_8	Manual Grinding	T_5, T_6 closed, Day	3.0	10'000	71'890
S_9	Manual Grinding	T_5, T_6 closed, Day	3.0	6’000	71'890
S_{10}	Manual Grinding	T_5, T_6 closed, Day	3.0	6’000	71'890
S_{11}	Manual Grinding	T_5, T_6 closed, Day	3.0	6’000	71'890
S_{12}	Manual Grinding	Night	3.0	10'000	68'662
S_{13}	Manual Grinding	Night	3.0	10'000	60'159
S_{14}	Manual Grinding	Night	3.0	10'000	67'940
S_{15}	Manual Grinding	Night	3.0	10'000	67'940
S_{16}	Manual Grinding	Night	3.0	10'000	68'662
S_{17}	Manual Grinding	Night	3.0	10'000	68'662
S_{18}	Manual Grinding	Night	3.0	10'000	67'940
S_{19}	Manual Grinding	Night	3.0	10'000	68'662
S_{20}	Manual Grinding	Night	3.0	10'000	68'662
S_{21}	Manual Grinding	T_{10} closed, Weekend	3.0	10'000	64'627
S_{22}	Manual Grinding	T_9 closed, Weekend	3.0	6’000	60'159
S_{23}	Manual Grinding	Night	3.0	10'000	68'301
B_{16}	Renewal	T_1, T_2, T_3, T_4, T_{11} closed, Day	72.0	3'200'000	10'499'196
B_{28}	Renewal	T_5, T_6 closed, Day	72.0	3'825'000	54'499'698
Total				**8’639’441**	**66’927606**

Table 40. Traffic states of the intervention program for the exhaustive search

Closed track segments	Time window	Without budget limitation	
		Duration [h]	Costs [euro]
T3	Day	4.5	22'429
T4	Day	4.5	22'429
T5	Day	4.6	31'570
T6	Day	4.6	31'570
T1, T2, T3, T4, T11	Day	72.0	3'000'276
T5, T6	Day	72.0	2'988'789
T9	Weekend	3.0	0
T10	Weekend	3.0	2'639
Different track segments	Night	35.8	0
Total user costs:			**6'097'063**

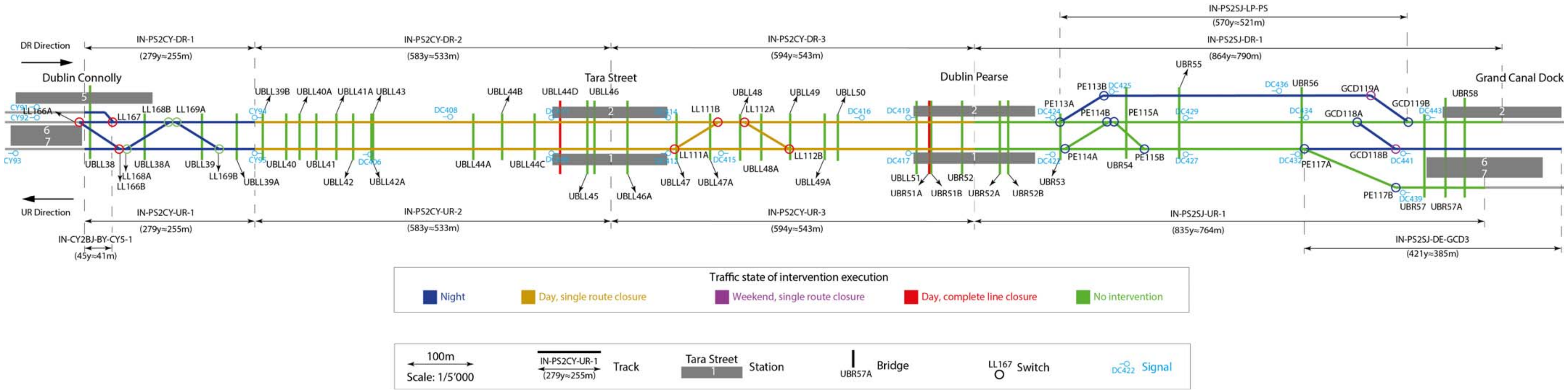

Figure 15. Intervention program by exhaustive search without a budget limitation

Annex D Results of decision rules

Table 41. Intervention program developed by the decision rules without a budget limitation

ID	Intervention	Traffic state	Duration [h]	Owner costs [Euros]	Risk reduction [Euros]
T_1	Tamping	Night	0.6	1'913	56'492
T_2	Tamping	Night	0.6	1'913	56'492
T_3	Track Renewal	T_3 closed, Day	4.5	397'405	57'616
T_4	Track Renewal	T_4 closed, Day	4.5	397'405	57'616
T_5	Track Renewal	T_5 closed, Day	4.6	323'889	58'402
T_6	Track Renewal	T_6 closed, Day	4.6	323'889	58'402
T_9	Ballast Cleaning	Night	4.4	990	180'684
T_{10}	Ballast Cleaning	Night	3.2	732	33'104
T_{11}	Tamping	Night	0.1	308	4'551
S_1	Manual Grinding	T_1, T_2, T_3, T_4, T_{11} closed, Day	3.0	10'000	92'472
S_2	Manual Grinding	T_1, T_2, T_3, T_4, T_{11} closed, Day	3.0	6'000	92'472
S_3	Manual Grinding	T_1, T_2, T_3, T_4, T_{11} closed, Day	3.0	6'000	92'472
S_8	Manual Grinding	T_5, T_6 closed, Day	3.0	10'000	71'890
S_9	Manual Grinding	T_5, T_6 closed, Day	3.0	6'000	71'890
S_{10}	Manual Grinding	T_5, T_6 closed, Day	3.0	6'000	71'890
S_{11}	Manual Grinding	T_5, T_6 closed, Day	3.0	6'000	71'890
S_{12}	Manual Grinding	Night	3.0	10'000	68'662
S_{13}	Manual Grinding	Night	3.0	10'000	60'159
S_{14}	Manual Grinding	Night	3.0	10'000	67'940
S_{15}	Manual Grinding	Night	3.0	10'000	67'940
S_{16}	Manual Grinding	Night	3.0	10'000	68'662
S_{17}	Manual Grinding	Night	3.0	10'000	68'662
S_{18}	Manual Grinding	Night	3.0	10'000	67'940
S_{19}	Manual Grinding	Night	3.0	10'000	68'662
S_{20}	Manual Grinding	Night	3.0	10'000	68'662
S_{21}	Manual Grinding	Night	3.0	10'000	64'627
S_{22}	Manual Grinding	Night	3.0	10'000	60'159
S_{23}	Manual Grinding	Night	3.0	10'000	68'301
B_{16}	Renewal	T_1, T_2, T_3, T_4, T_{11} closed, Day	72.0	3'200'000	10'499'196
B_{28}	Renewal	T_5, T_6 closed, Day	72.0	3'825'000	54'499'698
Total				**8'805'385**	**66’927606**

Table 42. Intervention program developed by decision rules with a budget limitation of 4 Mio. Euros

ID	Intervention	Traffic state	Duration [h]	Owner costs [Euros]	Risk reduction [Euros]
T_1	Tamping	Night	0.6	1'913	56'492
T_2	Tamping	Night	0.6	1'913	56'492
T_3	Track Renewal	T_3 closed, Day	4.5	397'405	57'616
T_4	Track Renewal	T_4 closed, Day	4.5	397'405	57'616
T_9	Ballast Cleaning	Night	4.4	990	180'684
T_{11}	Tamping	Night	0.1	308	4'551
B_{16}	Renewal	T_1, T_2, T_3, T_4, T_{11} closed, Day	72.0	3'200'000	10'499'196
Total				**3'999'932**	**10'912'647**

Table 43. Traffic states of the intervention program for the decision rule approach

Closed track segments	Time window	Without budget limitation		With budget limitation	
		Duration [h]	Costs [euro]	Duration [h]	Costs [euro]
T3	Day	4.5	22'429	4.5	22'429
T4	Day	4.5	22'429	4.5	22'429
T5	Day	4.6	31'570	-	-
T6	Day	4.6	31'570	-	-
T1, T2, T3, T4, T11	Day	72.0	3'000'276	72.0	3'000'276
T5, T6	Day	72.0	2'988'789	-	-
Different track segments	Night	41.8	0	5.6	0
Total user costs:			**6'097'063**		**3'045'135**

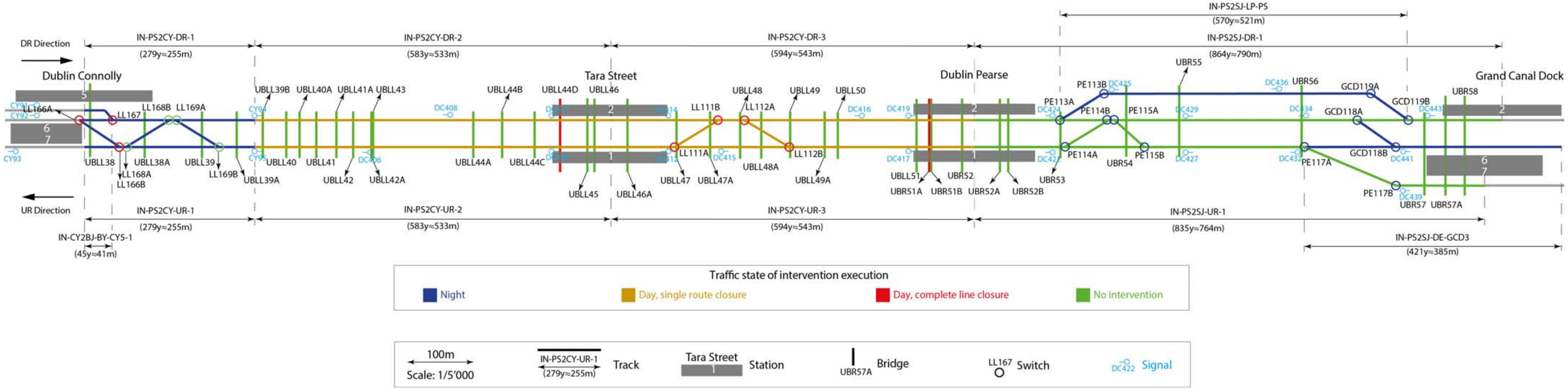

Figure 16. Intervention program by decision rules without a budget limitation

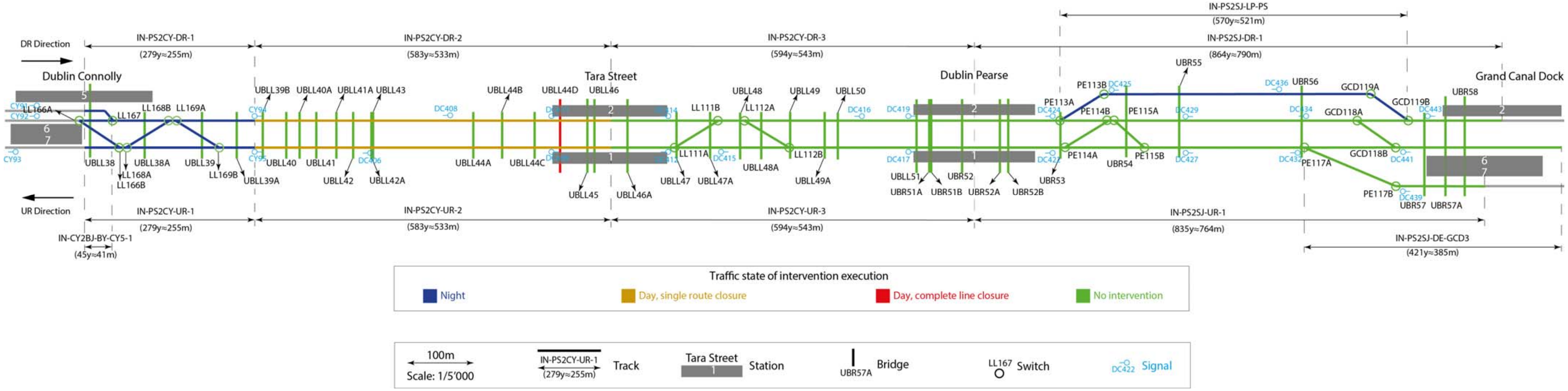

Figure 17. Intervention program by decision rules with a budget limitation of 4 Mio. Euros

Annex E Results of network flow model

Table 44. Intervention program developed with the network flow model without a budget limitation

ID	Intervention	Traffic state	Duration [h]	Owner costs [Euros]	Risk reduction [Euros]
T_1	Tamping	Night	0.6	1'913	56'492
T_2	Tamping	Night	0.6	1'913	56'492
T_3	Track Renewal	T_3 closed, Day	4.5	397'405	57'616
T_4	Track Renewal	T_4 closed, Day	4.5	397'405	57'616
T_5	Track Renewal	T_5 closed, Day	4.6	323'889	58'402
T_6	Track Renewal	T_6 closed, Day	4.6	323'889	58'402
T_9	Ballast Cleaning	Night	4.4	990	180'684
T_{10}	Ballast Cleaning	Night	3.2	732	33'104
T_{11}	Tamping	Night	0.1	308	4'551
S_1	Manual Grinding	T_1, T_2, T_3, T_4, T_{11} closed, Day	3.0	10'000	92'472
S_2	Manual Grinding	T_1, T_2, T_3, T_4, T_{11} closed, Day	3.0	6'000	92'472
S_3	Manual Grinding	T_1, T_2, T_3, T_4, T_{11} closed, Day	3.0	6'000	92'472
S_8	Manual Grinding	T_5, T_6 closed, Day	3.0	10'000	71'890
S_9	Manual Grinding	T_5, T_6 closed, Day	3.0	6'000	71'890
S_{10}	Manual Grinding	T_5, T_6 closed, Day	3.0	6'000	71'890
S_{11}	Manual Grinding	T_5, T_6 closed, Day	3.0	6'000	71'890
S_{12}	Manual Grinding	Night	3.0	10'000	68'662
S_{13}	Manual Grinding	Night	3.0	10'000	60'159
S_{14}	Manual Grinding	Night	3.0	10'000	67'940
S_{15}	Manual Grinding	Night	3.0	10'000	67'940
S_{16}	Manual Grinding	Night	3.0	10'000	68'662
S_{17}	Manual Grinding	Night	3.0	10'000	68'662
S_{18}	Manual Grinding	Night	3.0	10'000	67'940
S_{19}	Manual Grinding	Night	3.0	10'000	68'662
S_{20}	Manual Grinding	Night	3.0	10'000	68'662
S_{21}	Manual Grinding	T_{10} close, Weekend	3.0	10'000	64'627
S_{22}	Manual Grinding	T_9 close, Weekend	3.0	6’000	60'159
S_{23}	Manual Grinding	Night	3.0	10'000	68'301
B_{16}	Renewal	T_1, T_2, T_3, T_4, T_{11} closed, Day	72.0	3'200'000	10'499'196
B_{28}	Renewal	T_5, T_6 closed, Day	72.0	3'825'000	54'499'698
Total				**8’639’441**	**66’927606**

Table 45. Intervention program developed with the network flow model with a budget limitation of 4 Mio. Euros

ID	Intervention	Traffic state	Duration [h]	Owner costs [Euros]	Risk reduction [Euros]
T_1	Tamping	Night	0.6	1'913	56'492
T_2	Tamping	Night	0.6	1'913	56'492
T_3	Track Renewal	T_3 closed, Day	4.5	397'405	57'616
T_4	Track Renewal	T_4 closed, Day	4.5	397'405	57'616
T_9	Ballast Cleaning	Night	4.4	990	180'684
T_{11}	Tamping	Night	0.1	308	4'551
B_{16}	Renewal	T_1, T_2, T_3, T_4, T_{11} closed, Day	72.0	3'200'000	10'499'196
Total				**3’999’932**	**10’912’647**

Table 46. Traffic states of the intervention program for the network flow model

Closed track segments	Time window	Without budget limitation		With budget limitation	
		Duration [h]	Costs [euro]	Duration [h]	Costs [euro]
T3	Day	4.5	22'429	4.5	22'429
T4	Day	4.5	22'429	4.5	22'429
T5	Day	4.6	31'570	-	-
T6	Day	4.6	31'570	-	-
T1, T2, T3, T4, T11	Day	72.0	3'000'276	72.0	3'000'276
T5, T6	Day	72.0	2'988'789	-	-
T9	Weekend	3.0	0	-	-
T10	Weekend	3.0	2'639	-	-
Different track segments	Night	35.8	0	5.6	0
Total user costs:			**6'097'063**		**3'045'135**

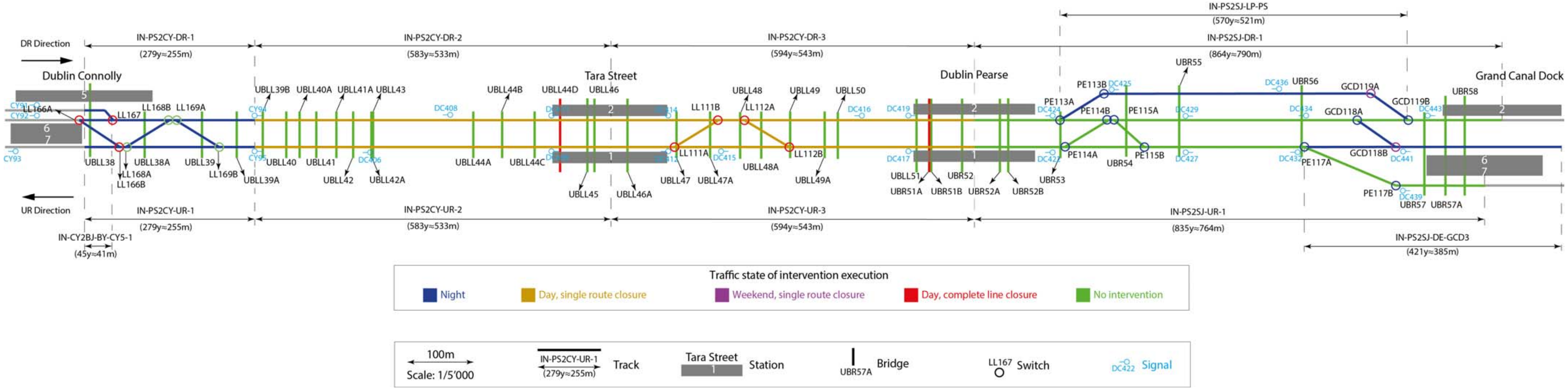

Figure 18. Intervention program of the network flow model without a budget limitation

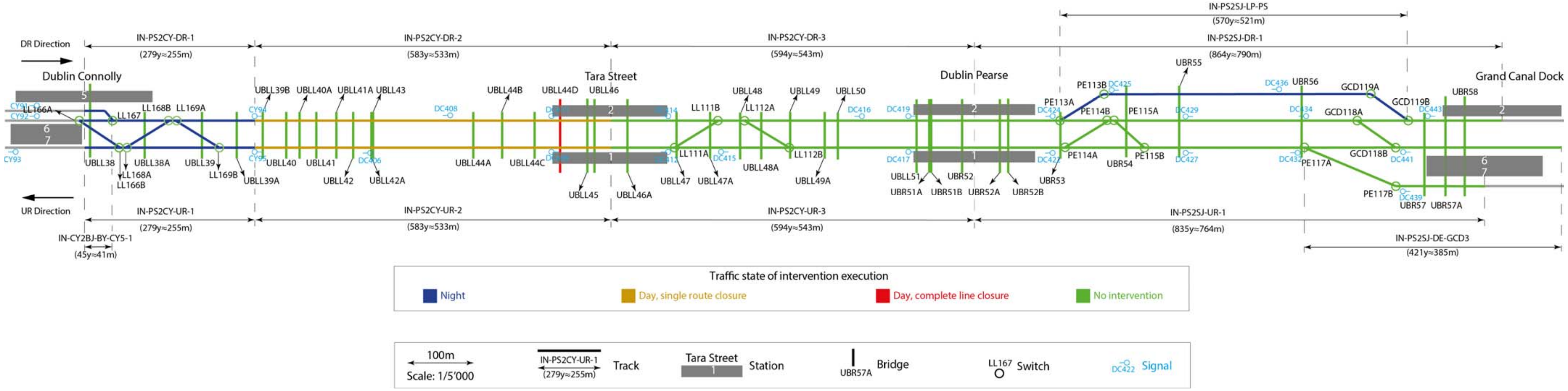

Figure 19. Intervention program of the network flow model with a budget limitation of 4 Mio. Euros